THE UNIVERSITY OF ALBERTA

EFFECTS OF LOW FERTILITY ON THE STABILIZATION PROCESS OF A NON-CATASTROPHICALLY DISTORTED AGE DISTRIBUTION

by

MOHAMMAD ASHRAF KHAN KAYANI ©

A THESIS

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To my Bhai Jan, Lt. Colonel Afzal Kayani,
for without his love, support,
and direction I would not have
come this far.

ABSTRACT

The present study examines the effects of low and monotonically declining fertility on the stabilization process of a human population. It may, also, be classified as an extension of Coale's work on the stabilization process of a human population. The basic approach used in the study is that of Keyfitz's projection matrix approach which involves the method in which population dynamics are depicted as a sequence of matrices.

A stable population is defined as a closed population with constant fertility and mortality schedules. In a closed population, the interaction between the constant schedules of fertility and mortality over a period of time produces one and only one age distribution which is independent of its initial age structure.

A new way of visualizing the stabilization process of a population has been proposed in the present study. We have found that the notion of slope(s) between two successive age groups of an age distribution in determining the stability of a population is as reliable as any other criterion.

It has been observed that the mean of the slope distribution is positively related to the level of fertility when the process of stabilization is completed.

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The variance of the slope distribution indicates changes in the age of a population. High fertility schedules overcome oscillations in the slope distributions more quickly than low fertility schedules.

The distance between initial and stable age distributions, both in terms of proportions and time, is critically reviewed. We have found that the index of dissimilarity is not a proper indicator of the temporal distance (in terms of proportions). However, it is found that the index of dissimilarity is a useful measure of the aging process of a population.

Length of the process of stabilization is found to be related with (a) level of fertility, (b) degree of consistency between the fertility schedule and initial age distribution, and (c) the interaction between level of fertility and the degree of consistency (b). It is observed that high fertility is negatively related with the length of the process. We also found that fertility schedules that are very different from those implied by the initial age distribution take a much longer time to stabilize the proportionate age distributions than the schedules more consistent with the initial age distribution. If the degree of consistency is less, between the fertility schedule and the initial age distribution, and the fertility level is very low, the length of the process will be longer than the two instances we have mentioned above.

The process of stabilization and the social aspects of achieving a stabilized population are discussed and the advantages of lower fertility schedules, in the long run, over the higher fertility schedules are demonstrated. It is argued that whereas the utility of stable population concept is important in estimating population growth, size, distributions, and the vital processes in developing countries with scanty data, the concept of stable population is equally important for the developed countries in understanding their socio-economic problems.

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TABLE OF CONTENTS

CHAPTER		PAGE

LIST OF TABLES

TABLE PAGE

LIST OF FIGURES

CHAPTER I

MODELS OF POPULATION GROWTH

In this chapter the general background of the study is described. A brief review of literature on the models regarding mortality, fertility and population growth is provided. Owing to its very special nature, the migration component of population growth is not discussed in our analysis. In Chapter II, details of the present research are provided.

There are two components of natural growth of population, namely, fertility and mortality. A population which changes only through fertility and mortality is termed a closed population whose growth and the components of growth are extensively analysed in the literature through models. The contemporary developments in model building discussed by Keyfitz (1971), and independent reviews of most of the models provided by Sheps et al (1969) and Krishnan (1971) have been drawn on because of their relevance to the study.

MODELS IN MORTALITY STUDIES

Mortality has always been of major concern to both the lay and specialist persons. Most individuals and societies approve, implicitly or explicitly, the attempts to enhance the chances of human survival. Such a consensus on the importance of mortality studies perhaps provided an impetus to the social scientists of the past to devote their attention to the determinants and the consequences of mortality. One indication of this long-standing concern with mortality is the systematic and conceptual framework on mortality developed as early as 1662 by Graunt. Based on Graunt's framework, Halley constructed the first empirical life table in 1693.

Since 1693, much attention has been paid to developing life tables with increasing methodological sophistication. Due to the regularities in the age-specific mortality patterns in almost all societies, the model life tables by the United Nations (1955 and 1968), Coale and Demeny (1966), Ledermann (1969), and Carrier and Hobcraft (1971) have become benchmarks in contemporary mortality studies. The major assumption in the model life tables is that the age-sex differentials of mortality are interrelated. The powerfulness of the model life tables lies in the fact that from mortality knowledge for one age-group the mortality rates for all other age groups (except infant mortality rates) can be estimated.

As mortality has declined in the past almost all over the world, there is a growing interest in constructing mortality tables by causes of death (Preston, Keyfitz and Schoen, 1972). The regularities in the mortality patterns and the degree of accuracy achieved by the model life tables have resulted in fairly reasonable mortality projections by the generation and the time trend methods (Spiegelman, 1968, pp. 153-63).

MODELS OF FERTILITY AND POPULATION GROWTH

Models of fertility and population growth may be discussed under a variety of topics. A bird's-eye view of some of these models is presented and arranged as stochastic models, model fertility tables approach, mathematical models, theoretical frameworks, and the stable population model.

Stochastic Models

The stochastic models are available both at the micro- and macro-level of analysis. Applications of mathematical and statistical knowledge and the use of computers are the core of contemporary stochastic models. The works of Louis Henry (1953, 1957, 1961, and 1964), Brass (1958), Singh (1964), and Perrin and Sheps (1964) are widely mentioned in the literature. Henry's works deal with the 'natural' fertility and are confined mainly to the investigation of the effects of physiological variations

on human reproduction. His models of 'natural' fertility and family building deal with the problems of measurement of reproductive performance of married females. The models investigate the 'natural' fertility variations, the role of biological determinants of reproduction, the age at marriage, etc..

Brass's (1958) model deals with the probability distribution of births to mothers with completed fertility. Singh's (1964) model relates with the generalization that the probability distributions of the number of conceptions with fixed exposure follows a Poisson distribution.

Perrin and Sheps (1964) applied renewal theory to the study of human reproduction. Perrin and Sheps' model has been supplemented from time to time by Sheps (1964; 1966a; and 1966b). In the model, a woman entering a sexual union is exposed to the risks of conception and the termination of pregnancy is allowed to be in one of the several forms. The models by Henry, Brass, Singh, and Perrin and Sheps are micro in nature.

Model Fertility Tables Approach

Bourgeois-Pichat (1965) has made use of certain sociological and physiological variables at the macro-level. His approach is analytical in nature supplemented by empirical findings. In the prepartion of model fertility tables, he classified the world into five regions based on five patterns of marriage. The combination of factors such as

coital frequency, infertility, foetal mortality, are used to get different possibilities of natural fertility. He arrives at the figure of 16,170 possibilities of natural fertility by using five nuptiality patterns, seven infertility patterns, fourteen lengths of infertile periods, coital frequency as low as two and as high as thirty per cycle, and three lengths of ovum. However, he does not consider in his model fertility tables the effects of variables such as the use of contraception, changes in social norms and values, and so forth.

Among others, the contributions by Romaniuk (1973) and Coale and Trussell (1974) are important to mention. Romaniuk's (1973) model, which is specifically designed for fertility projections, makes use of three simple demographic parameters, namely, level of fertility and mean and mode ages of fertility. His model is based on conventional frequency distributions such as Pearsonian curves. Coale's and Trussell's (1974) model fertility tables are based on age patterns of fertility. Their model is flexible enough to incorporate a combination of demographic factors such as age at marriage, mortality, etc..

Mathematical Models

Mathematical models of population growth have also been emphasized in demographic literature. Contributions by Keyfitz (1964, 1965, and 1968) and Goodman (1968) cannot be overlooked when one is dealing with the population models.

The use of matrix algebra is the basic tool of their approach where a closed population is assumed and the past fertility and mortality experiences of a population are used as the basis of their matrix approach. These mathematical models are mainly used in preparing population projections.

Theoretical Framework

To clarify, the usefulness of models lies in predictions and "the accuracy of predictions is a test of the models" (Keyfitz, 1971, p. 571). Since mortality patterns in most nations can be known with fairly reasonable reliability while fertility estimates have not reached that level of sophistication, it can be stated that the stochastic and mathematical models of fertility have failed to prove as accurate as mortality models. For the understanding of fertility phenomenon, demographers with sociological interests have suggested theoretical frameworks for the analysis of fertility behavior. The well-known frameworks are: the institutional approach by Davis and Blake (1956), the interactional approach by Hill, Stycos, and Back (1959), and the normative approach by Freedman (1961-62).

Davis and Blake recognize the role of certain "intermediate" variables, through which and only through which cultural factors can affect fertility. It is difficult to study the direct impact of cultural factors or "conditioning factors" on fertility, but there are others which serve as a means of analysing these social and

cultural factors. These factors are termed "intermediate variables" and are further classified as "intercourse variables," "conception variables," and "gestation variables." In all, there are eleven intermediate variables through which social and cultural conditions must operate in order to influence fertility positively or negatively.

Davis' and Blake's model suggests that the effect of intermediate variables on the fertility of a society is the product of institutional mechanisms of that society. They have arranged all the intermediate variables according to their expected values in a pre-industrial society into four categories, namely, usually high values, high or low values, usually low values, and the indeterminate.

Hill's, Stycos' and Back's (1959) interactional approach, though applied only to Puerto Rican society and at individual levels, presents a scheme where the influences of key reference groups and demographic factors on fertility are felt through general value systems, family action possibilities and attitudes, knowledge and practice of family planning. In their scheme, the informational and attitudinal attributes, general value system and specific family size preference interact with each other to produce family action possibilities. The family action possibilities further interact with the variables of effective family planning so as to produce a certain level of fertility.

The normative approach by Freedman (1961-62) is an elaboration of the institutional approach with the emphasis on norms about family size in a group or a society. Freedman starts with the assumption that social norms in a society develop concerning a particular fertility level which affects factors such as population growth rate and age structure. He explicates that the social norms and the norms about each of the intermediate variables together produce a fertility level. Mortality, net migration, and the values about the intermediate variables are important aspects of social organization which support the norms about family size by providing social sanctions related to the number of children per family unit. Environmental factors such as famine, venereal diseases, which affect the norms about the family size are taken into account. In his presidential address to the annual meeting of the Population Association (1965), Freedman states the conditions under which rapid fertility decline could be possible. These conditions are: (1) significant social development, (2) low mortality, (3) smaller family size preferences, (4) dissemination of family planning ideas, and (5) the availability of effective contraceptives.

These frameworks are useful in understanding the dimensions of fertility. But they face numerous problems of measurement of social phenomenon and therefore are limited in their practicality.

Stable Population Model

Another class of models is known as deterministic models of population growth. The development of deterministic models may well be traced back to Dublin and Lotka (1925) and Lotka (1939, 1940). Lotka introduced the concept of stable population as a special case of Malthusian population (U.N., 1968, p. 1). In a Malthusian population, mortality and age-structure are constant (U.N., 1968, p. 1). In a stable population, fertility and mortality remain constant and interaction between fertility and mortality produces a unique age-distribution over a period of time. The unique age-distribution, thus sought, is different, except in extreme cases, from the initial age-distribution and is termed the stable age-distribution (Coale, 1956 and 1957; McFarland, 1969; Bourgeois-Pichat, 1971).

The central idea in the stable population theory is that: (a) the population is closed to migration; (b) age-specific fertility and mortality schedules remain constant; and (c) the interaction between constant fertility and mortality schedules, over a long period of time, produces a stable age-distribution which is independent of the initial age-distribution. These characteristics of the stable population are also called the 'intrinsic characteristics" which in fact are the characteristics of initial conditions of fertility and mortality (U.N., 1968, p. 8). Mathematical proof of the stable population theorem is

given by Coale (1972) and Lopez (1961). The stable population theory deals with both weak and strong ergodic properties of a population. A brief description of weak and strong ergodic properties of a population is provided in the section on 'Research Problem' of Chapter II.

One of the major features of stable population theory is the determination of the effects of changes in mortality and fertility schedules on the age-distribution. Before the exploration of the theory, it was thought that high mortality produces a young population and low mortality produces an old population.

Coale (1956) is the first to demonstrate that the aging process of a population is largely dependent on fertility and not on mortality, except in extreme cases. A high fertility schedule produces a younger population, while a low fertility schedule produces an older population.

The mortality effects on the age-distribution are significant when there are extreme differences in the mortality levels (U.N., 1968, p. 105). For example, if the fertility schedule is fixed and mortality levels vary, say, from life expectancy of 20 years at birth to life expectancy of 75 years at birth, the resultant age-distributions would be dissimilar. Coale (1972, pp. 152-64) has also demonstrated the effects of age-specific mortality changes on the age-distribution. In his analysis, Coale assumed a fixed fertility schedule and a specified monotonic time

pattern of mortality change. It is shown that the effects of mortality decline on the age-distribution are dependent on the age-groups that experience the decline. For example, the effects of mortality decline in the age-distribution differ when mortality changes occur only to persons under age five from those when mortality decline is experienced only by older age-groups. The age-selective changes in mortality pose serious problems when one is estimating demographic parameters using the stable population model. Demeny (1965) has pointed out some of the biases of the estimates of vital rates for the populations that are in the process of de-stabilization. The Coale principle still holds good in empirical proportions as they occur in real life.

The stable population theory is useful in determining the necessary relationships among demographic processes under given conditions. The theory is considered to be the most powerful tool of analysis yet available in demographic literature. One of the criticisms of the theory, however, is that the stable population concept is highly abstract in nature. It is argued that no population has been found to have mortality and fertility schedules constant over a long period of time. To rectify the inadequacy of the stable population model, the concepts of semi-stable and quasi-stable populations have been developed to help demonstrate the practicality of the stable population

model in the real populations (U.N., 1968, p. viii). These concepts are described below.

The populations where fertility and mortality schedules are constant over a shorter period of time and the age-structure is either constant or changes very slightly are known as semi-stable populations. The major distinctions between a stable population and a semi-stable population is in terms of the time lag between the initial and the final age-distribution. In a stable population the time lag between initial and stable age-distribution is very long, while in a semi-stable population the time lag is very short, or usually zero. In the light of this distinction, most populations of the world may be characterized as semi-stable populations (U.N., 1968, pp. vii-ix).

In a quasi-stable population, fertility remains constant and the risks of mortality diminish continuously. Since age-structure is largely determined by fertility [except for extreme mortality changes (Coale, 1956)], the age-structure is presumed to be constant. Most of the developing nations of today in which mortality has declined in the past and fertility remained constant are classified as quasi-stable populations. The importance of the quasi-stable population concept lies in the fact that one of the assumptions in the stable population theory (that concerning mortality) can be relaxed without effects on the age-distribution. The extent to which changes do occur are

given in the U.N. Manual IV (1967, p. 47).

It may be added that both the concepts of semi-stable population and quasi-stable population reflect the operational power of the stable population theory. One indication of this operational power of the stable population theory is that it has enabled demographers to prepare reliable estimates even when the data are scanty (Beaujot, 1972; Coale, 1963; Coale and Hoover, 1958; Demeny, 1965; Demeny and Shorter, 1968; Krotki, 1963 and 1969a; Krotki and Thakur, 1971; Romaniuk, 1967; Romaniuk and Piche, 1972; United Nations, 1967).

CHAPTER II

RESEARCH PROBLEM, BASIC APPROACH AND DATA

This chapter consists of three parts. The first part deals with the statement of the problem under investigation. Part two deals with the basic analytical approach, while part three discusses the data used in this study.

RESEARCH PROBLEM

As stated in Chapter I, the stable population theory deals with two properties of a population, namely, strong and weak ergodic properties. A population with strong ergodic characteristics tends to drift from its past age-distribution over a period of time towards reaching a 'limiting stable form' in its age-distribution (Lopez, 1961 and 1967). Again, in strong ergodicity, fertility and mortality conditions remain unchanged.

The major area of exploration in the strong ergodicity theorem is the analysis of the process of convergence from an initial population to its stable form. Coale (1968) has dealt with this topic using an approach that involves the method in which sequence of births is defined by means of integral equation.

With weak ergodic characteristics a population forgets its past age-distribution even if fertility and mortality schedules are changing. Hajnal (1958), Lopez (1961) and McFarland (1969) have gone into a detailed analysis of weak ergodicity. Their analysis shows that when migration does not affect fertility and mortality conditions of a society, the non-sustained migration would have only a transient effect on age-distribution of a population.

Some of the arguments of weak ergodicity are applicable to strong ergodicity except that the ultimate age-distribution for strong ergodicity is compactly specified (Keyfitz, 1968, p. 90).

The present attempt aims at analysing the process of stabilization of a given population; that is, how a population with given fertility and mortality schedules attains stability. Our focus of analysis would be to identify fertility effects on the initial age-distributions. Such an analysis will help in understanding some of the strong ergodic properties of the stable population theory. The approach used in the present study is based upon the method in which population dynamics are defined in terms of the product of a sequence of matrices.

BASIC APPROACHES

There are two approaches through which the analysis of stable population is possible: these approaches are the

integral equation approach and the projection matrix approach. In the integral equation approach the sequence of births is calculated as the sum of real exponential terms and a series of relatively diminishing oscillatory terms (Coale, 1972, p. 64).

In the integral equation approach the birth and death rates vary with age which is treated as a continuous variable, while in the projection matrix approach the birth and death rates are calculated for discrete age intervals (usually five years). The integral equation approach is quite complex in treatment and laborious in computations. On the other hand, the computational part in the projection matrix approach is less laborious owing to the availability and use of electronic computers. However, the projection matrix approach is thought to reveal little about the factors that are responsible for a slow or fast convergence of a population to its stable form. This latter aspect is adequately handled in the integral equation approach. It has been demonstrated that the integral equation approach is useful to show ". . . how salient features of the process of convergence are determined by certain characteristics of the fertility and mortality schedules on the one hand, and certain properties of the initial age-distribution on the other" (Coale, 1972, p. 62). The projection matrix approach, though lacking in the exactness of the integral approach, provides analytical insights into the process that the

integral method may fail to reveal.

As stated earlier, understanding of advanced mathematical tools is necessary for the integral equation approach. The projection matrix approach has certain advantages over the integral one:

(a) Less mathematical depth is required for the analysis,

(b) The basic quantities that appear in the projection matrix approach are more closely related to the quantities that are used in demographic practice as compared to the quantities that appear in the integral equation approach.

Keyfitz (1968, pp. 41-73) has highlighted the analytical aspects of stable populations by means of projection matrix approach. He suggests that with given fertility and mortality schedules, the M matrix be raised to a number of powers till the stability conditions are satisfied. The method of raising the M matrix to a number of powers is useful in many respects, but it does not disclose all the intermediate steps through which a population achieves stability.

The author is not aware of attempts to explain all or some of the features of the process of stabilization of a human population by means of the projection matrix approach. The present study is a step in that direction; it analyses some aspects of the stable population as well

as the process by which a human population converges to its stable form using projection matrix approach.

PROJECTION MATRIX APPROACH

In a closed population, population dynamics are characterized only by fertility and mortality. The mortality is applicable to persons of all ages, while fertility is applied only to people of the selected ages. Those who die are excluded from the population and those born are included. There are also age and sex differentials of fertility and mortality. In the computations of simple population projections the fertility schedules are generally applied to the female population at risk. Since there are mortality differentials by sex, risks of mortality are applied to each sex separately (Spiegelman, 1968).

In population projections where fertility and mortality are assumed to be constant, five distributions are needed for population projections. The five distributions are:

(a) Initial female age-distribution

(b) Initial male age-distribution

(c) Fertility schedule (female dominant)

(d) Female mortality schedule

(e) Male mortality schedule

The age interval of the distributions can be fixed as desired and also the time interval for population

projections. For example, one may use projection matrix approach for age-distributions with single year of age interval and different (other than one year) time interval for projections. However, age intervals in mortality and fertility schedules should be the same. It is convenient to set equal time and age intervals in population projections. A simple and less laborious example may be population projections by five-year age intervals and by five-year time intervals.

In our analysis, five-year time intervals and five-year age-groups are used.

The total number of births during five years of population projection is calculated by using a set of linear, first order, homogeneous difference equations with constant coefficients by the following formula (Keyfitz, 1968, p. 31):

$$K_0^{(t+5)} = \frac{5\,{}_5L_0}{l_0} \left\{ \left[\frac{K_{10}^{(t)} + K_{10}^{(t+5)}}{2} \right] F_{10} + \left[\frac{K_{15}^{(t)} + K_{15}^{(t+5)}}{2} \right] F_{15} + \ldots + \left[\frac{K_{45}^{(t)} + K_{45}^{(t+5)}}{2} \right] F_{45} \right\} \qquad 2.1$$

Equation 2.1 can further be simplified as:

$$K_0^{(t+5)} = \frac{5}{2} \times \frac{{}_5L_0}{l_0} \left\{ \left[K_{10}^{(t)} + K_{10}^{(t+5)} \right] F_{10} \right.$$

$$+ \left[K_{15}^{(t)} + K_{15}^{(t+5)} \right] F_{15} + \ldots$$

$$\left. + \left[K_{45}^{(t)} + K_{45}^{(t+5)} \right] F_{45} \right\} \qquad 2.2$$

where $K_x^{(t)}$ = female population age x at time t

F_x = fertility rates for females age x

t = five-year time interval

$\frac{{}_5L_0}{l_0}$ = factor allowing for the children born during five years to survive to the end of the period.

The equations 2.1 and 2.2 give us the total number of births to females aged 10-49 during five years of projections. The total number of births thus sought is multiplied by male and female proportions at birth to get the total number of children by sex.

The expression $\frac{5}{2}\left[K_x^{(t)} + K_x^{(t+5)}\right]$ is used to get an estimate of the number of women-years of exposure over the period. This expression is multiplied by age-specific fertility rates to get the number of births for the females of each age-group.

As the fertility rates used are period rates and are fixed while the age-groups are allowed to move upward in the age pyramid during the five-year projection interval, Keyfitz (1968) suggests that by taking the average of the same age-groups at the time (t) and (t+5) reliable estimates of births to that age-group may be obtained.

The total number of male and female births during the five-year time interval are multiplied by the sex-specific survival ratios to form the sex-specific 0-4 age-groups at (t+5). The survival ratio used in the projections is $\frac{{}_5L_0}{l_0}$. The population of age-groups other than 0-4 at time t+5 is projected by the formula:

$$\frac{{}_5L_x}{{}_5L_{x-5}} K_{x-5}^{(t)} = K_x^{(t+5)} \qquad 2.3$$

As the age-group 0-4 for each sex is established, the entire set may compactly be written as:

$$L \left\{\overline{K}^{(t)}\right\} = \left\{\overline{K}^{(t+5)}\right\},$$

where $\left\{\overline{K}^{(t)}\right\}$ is vertical vector of sex-specific age-distribution at time t and L is the matrix of the coefficients of $K^{(t)}$ (Keyfitz, 1968, p. 31). For each sex, the L matrix and the vertical vectors can be written in the following form.

L Matrix   Vertical Vector

$$
\begin{bmatrix}
- & - & - & - & - \\
\frac{{}_5L_0}{l_0} & 0 & 0 & 0\ldots\ldots & 0 \\
0 & \frac{{}_5L_5}{{}_5L_0} & 0 & 0\ldots\ldots & 0 \\
0 & 0 & \frac{{}_5L_{10}}{{}_5L_5} & 0\ldots\ldots & 0 \\
\vdots & \vdots & & \vdots & \vdots \\
0 & 0 & 0 & 0 & \frac{{}_5L_{85}}{{}_5L_{80}}
\end{bmatrix}
\qquad
\begin{bmatrix}
K_x^{(t)} \\
K_x^{(t)} \\
\vdots \\
K_{85}^{(t)}
\end{bmatrix}
$$

The first row of L matrix has already been shown in equation 2.2. The multiplication of vertical vector and the L matrix projects the population by age from time (t) to (t+5) years for age-groups 5-9 to the last age-group. Age-group 0-4 is sought by the first row of L matrix or equation 2.2.

We have exactly followed Keyfitz's projection matrix approach in that we have raised the L matrix to the desired number of powers.

Since we are interested in understanding the process of convergence of a population to its stable form and not in mathematical properties of the matrices, we do not suggest to condense the matrices. By definition, a population is stable when the dominant root or the eigenvalue (λ_1) of the two matrices, that is, $K_x^{(t+5)}/K_x^{(t)}$ is exactly the same for all age-groups up to three decimal places. Use of λ_1 as a criterion for population stability has been

suggested and used by Keyfitz (1968, pp. 40-47). The computer program of population projections, which also computes other measures related to the age-distribution, is presented at the end of this chapter.

DATA

For an in-depth analysis of the stabilization process of human populations, one should select a variety of fertility and mortality schedules and initial age-distributions. Such a wide range of selection facilitates the possibility of making relatively more generalized statements. For example, one can select initial age-distributions that are distorted and undistorted, very young to very old; fertility schedules ranging from below replacement level fertility to the highest level of fertility yet experienced by human populations; and mortality schedules ranging from average life expectancy at birth of 20 years to 75 years. The interaction between fertility, mortality, and initial age-distribution during the process of stabilization, using the simulation technique and the projection matrix approach, will enable the researcher to specify the relationships between the demographic parameters. This type of analysis has three major drawbacks: (1) the number of combinations of fertility, mortality, and initial age-distribution will be too large to analyse; (2) the number of simulations will involve very high cost, labour and time; and (3) some of the

combinations of the schedules and initial age-distributions may be far from reality.

Other possible options for analysis range from selection of a particular combination of fertility and mortality schedules and an initial age-distribution to a fixed number of such combinations; a variety of fertility schedules with fixed mortality and an initial age-distribution; a variety of mortality schedules with fixed fertility and an initial age-distribution; and a variety of initial age-distributions with fixed fertility and mortality schedules. Each of these options have advantages and disadvantages over the others. Mostly it depends upon the researcher and the research problem as to the choice of options. For example, if one is to study the effects of mortality on the stabilization process of a population, one may select a variety of mortality schedules, one fertility schedule, and one initial age-distribution.

Since we are interested in evaluating the effects of fertility on the stabilization process of a population, we have made use of only one initial age-distribution and only one life table for each sex in the analysis. A variety of narrowly changing schedules of fertility are selected to examine the effects on the stabilization process and the final distributions. For this purpose, the U.S.A. 1963 population by sex is treated as the initial population. Note that the U.S.A. 1963 population distribution is distorted by

migration and recent declines in fertility. However, these distortions are not "catastrophic" to classify our selected initial age-distribution as unique in human populations (LeBras, 1969). The initial population and the corresponding life table values (L_x's) are taken from Keyfitz and Flieger (1968, p. 160). The initial population and the life table values are given in Table 2.1

Nine fertility schedules for the U.S.A. have been examined. Yearly age-specific fertility rates for the U.S.A. from 1960 to 1968 (U.S., DHEW, 1970) are presented in Table 2.2. In the analysis and discussion sections of subsequent chapters, each fertility schedule is named after the year for which age-specific rates constitute the respective schedule. For example, the 1960 fertility schedule means that the age-specific fertility rates of the U.S.A. in 1960 are applied. One of the limitations of the nine selected fertility schedules for the U.S.A., 1960-1968, is that there is a narrow range of monotonic decrease in fertility from 1960 to 1968. Discussion on changes in the U.S. fertility schedules of 1960-1968 is given in Chapter IV.

TABLE 2.1

Age-Sex-Specific Population and Life Table Values

for U. S. A., 1963

Age	Males	L_x for Males	Females	L_x for Females
Less than 1	2,090,000	97,837	2,002,000	98,364
1 - 4	8,482,000	387,593	8,175,000	390,386
5 - 9	10,203,000	483,302	9,872,000	487,032
10 - 14	9,170,000	482,132	8,866,000	486,201
15 - 19	7,790,000	480,046	7,677,000	485,177
20 - 24	6,015,000	476,281	6,289,000	483,647
25 - 29	5,328,000	472,002	5,512,000	481,721
30 - 34	5,538,000	467,595	5,747,000	479,205
35 - 39	5,996,000	461,873	6,285,000	475,549
40 - 44	5,943,000	453,414	6,270,000	470,118
45 - 49	5,481,000	440,127	5,731,000	462,050
50 - 54	5,008,000	419,100	5,232,000	450,086
55 - 59	4,305,000	387,821	4,557,000	432,921
60 - 64	3,587,000	345,066	3,942,000	408,146
65 - 69	2,865,000	288,747	3,377,000	372,220
70 - 74	2,282,000	222,740	2,806,000	322,196
75 - 79	1,488,000	155,007	1,917,000	255,557
80 - 84	759,000	91,898	1,068,000	172,925
85+	385,000	55,133	616,000	128,147
Total	92,715,000	-	95,941,000	-

Source:
Keyfitz, Nathan and W. Flieger, World Population: An Analysis of Vital Data, The University of Chicago Press, Chicago, 1968, p. 160.

TABLE 2. 2

Age Specific Fertility Rates for U. S. A., 1960 - 1968*

Age Group	1960	1961	1962	1963	1964	1965	1966	1967	1968
10 - 14	0.8	0.9	0.8	0.9	0.9	0.8	0.9	0.9	1.0
15 - 19	89.1	88.0	81.2	76.4	72.8	70.4	70.6	67.9	66.1
20 - 24	258.1	253.7	243.7	231.2	219.9	196.8	185.9	174.0	167.4
25 - 29	197.4	197.9	191.7	185.8	179.4	162.5	149.4	142.6	140.3
30 - 34	112.7	113.3	108.9	106.2	103.9	95.0	85.9	79.3	74.9
35 - 39	56.2	55.6	52.7	51.3	50.0	46.4	42.2	38.5	35.6
40 - 44	15.5	15.6	14.8	14.2	13.8	12.8	11.7	10.6	9.6
45 - 49	0.9	0.9	0.9	0.9	0.8	0.8	0.7	0.7	0.6
Total	3653.6	3629.0	3473.5	3333.2	3207.5	2928.0	2736.1	2572.6	2476.8

* Data for all years except 1967 are based on 50% sample of births. Data for 1967 are based on 20 to 50% sample of births.

Source:
U. S. A., Vital Statistics of the United States 1968, Vol. 1, Department of Health, Education and Welfare, Vol. 1 - Natality, Rockville, M. D.: 1970.

```
C     POPULATION PROJECTION PROGRAM BY ASHRAF K. KAYANI
C     AT THE START OF EACH AGE GROUP SMALL LX IS 100000.0
      DIMENSION A(20,100),B(20),C(20,100),D(20),AM(20)
     1,BM(20),DM(20),EM(20),FM(20),GM(20),HM(20),OM(20),PM(20),RPM(20),
     2DWM(20),RWM(20),RDM(20),WM(20),ZM(20),TM(20,100)
     1,CM(20)
      COMMON A,B,C,D,AM,DM,EM,FM,GM,HM,PM,WM,RDM,RWM,DWM
     1,I,M,ZM,TM,BM
C     READ DATA
C     A IS FEMALE POPULATION..B IS LX FOR FEM., C MALE POP..D LX FOR MALES
      DO 2 I=1,18
    2 READ(5,1)A(I,1),B(I),C(I,1),D(I)
    1 FORMAT(4(5X,F10.0))
C     AM=AGE SPECIFIC FERTILITY RATE
C     BM=AGE SPECIFIC LEGITIMATE FERTILITY RATE
C     CM= AGE SPECIFIC ILLEGITIMATE FERTILITY RATE
C     DM= AGE SPECIFIC MARITAL FERTILITY RATE
C     EM= STABLE % DISTRIBUTION OF MARRIED FEMALES
      READ(5,889)(AM(J),J=4,9)
      READ(5,889)(DM(J),J=4,9)
      READ(5,889)(BM(J),J=4,9)
      READ(5,888)(CM(J),J=4,9)
      READ(5,888)(EM(J),J=4,9)
  889 FORMAT(6F6.5)
  888 FORMAT(6F5.5)
C     KEYFITZ APPROACH
      CALL KEYFI
      STOP
      END
```

```
      SUBROUTINE KEYFI
      DIMENSION EE(20,100),A(20,100),CHIL(20,100),C(20,100),FF(20,100),
     1KK(20),KN(20),KKN(20),B(20),AM(20),D(20)
     1,ABC(20),BBC(20),CBC(20),BS(20),PA(20)
     1,ANM(20),AY(20),CY(20),EY(20)
     1,AAC(19,1),ABA(20),ACA(20),ADA(20)
     2,DM(20),EM(20),FM(20),GM(20),HM(20),PM(20),WM(20),RDM(20),RWM(20),DWM
     2DWM(20),ZM(20),TM(20,100),BM(20)
     2,ADM(20,100),CM(20)
     3,AKL(20),AJL(20),CKL(20),BKL(20)
      COMMON A,B,C,D,AM,DM,EM,FM,GM,HM,PM,WM,RDM,RWM,DWM
     1,I,M,ZM,TM,BM
C     CALC. TOTAL POP. BY AGE OF INPUT DATA FOR BOTH SEXES
      DO 99 I=1,18
   99 EE(I,1)=A(I,1)+C(I,1)
      DO 201 J=4,9
      TM(J,1)=A(J,1)*EM(J)
  201 ADM(J,1)=A(J,1)-TM(J,1)
      DO 100 M=2,45
C     FEMALE POPULATION PROJECTIONS
      DO 102 I=1,17
  102 A(I+1,M)=A(I,M-1)*(B(I+1)/B(I))
      DO 202 J=4,9
      TM(J,M)=A(J,M)*EM(J)
  202 ADM(J,M)=A(J,M)-TM(J,M)
      DO 101 J=4,9
  101 CHIL(J,M)=((A(J,M)+A(J,M-1))*AM(J))*2.5
      A(1,M)=(CHIL(4,M)+CHIL(5,M)+CHIL(6,M)+CHIL(7,M)+CHIL(8,M)+CHIL(9,M
     1))*.488475*.99045
C      MALE PROJECTIONS
      C(1,M)=(CHIL(4,M)+CHIL(5,M)+CHIL(6,M)+CHIL(7,M)+CHIL(8,M)+CHIL(9,M
     1))*.511525*.98808
      DO 103 I=1,17
  103 C(I+1,M)=C(I,M-1)*(D(I+1)/D(I))
C     CALCULATE TOTAL OF BOTH SEXES
      DO 104 I=1,18
      EE(I,M)=A(I,M)+C(I,M)
C     CALCULATE LAMBDA VALUE FOR     FEMALES
  104 FF(I,M)=A(I,M)/A(I,M-1)
C     PRINT RESULTS
      KAY=1963+(5*(M-1))
C      AGES PRINTED
      KN(1)=0
      KK(1)=5
      KKN(1)=4
      DO 105 I=2,18
      KN(I)=KN(I-1)+5
  105 KKN(I)=KN(I)+4
      WRITE(5,27)KAY
   27 FORMAT(1H1//2X,'POPULATION PROJECTIONS',10X,'YEAR',2X,I4//)
      WRITE(5,28)
   28 FORMAT(2X,'AGE-GROUP',10X,'FEMALE',10X,'MALE',10X,'BOTH SEXES',10X
     1,'LAMBDA FEMALES',2X,'% FEMALE DIST.',2X,'% INITIAL',2X,'2 INDEX'//)
     2/)
      AAC(1,1)=0.0
```

```
      ABC(1)=0.0
      ZM(1)=EE(1,1)
      BBC(1)=0.0
      CBC(1)=0.0
      DO 500 I=2,19
      AAC(I,1)=AAC(I-1,1)+A(I-1,1)
      ABC(I)=ABC(I-1)+A(I-1,M)
      BBC(I)=BBC(I-1)+C(I-1,M)
      ZM(I)=EE(I,1)+ZM(I-1)
  500 CBC(I)=CBC(I-1)+(A(I-1,M)+C(I-1,M))
      DO 30 I=1,18
      ABA(I)=(A(I,M)/ABC(19))*100.0
      ACA(I)=(A(I,1)/AAC(19,1)*100.0)
   30 ADA(I)=ABA(I)-ACA(I)
      DO 106 I=1,18
  106 WRITE(5,29)KN(I),KKN(I),A(I,M),C(I,M),EE(I,M),FF(I,M)
     1,ABA(I),ACA(I),ADA(I)
   29 FORMAT(4X,I2,'-',I2,9X,F11.0,4X,F11.0,6X,F12.0,9X,F9.6,10X,F5.2,7X
     1,F5.2,7X,F5.2)
      WRITE(5,501)ABC(19),BBC(19),CBC(19)
  501 FORMAT(1X,'TOTAL OF ALL AGES',F12.0,3X,F12.0,2X,F13.0)
      ANM(3)=12.5
      DO 703 J=4,9
      BS(J)=AM(J)*.488475*(B(J+1)/B(J))
      ANM(J)=ANM(J-1)+5.0
  703 PA(J)=ANM(J)*BS(J)
      RNN=BS(4)+BS(5)+BS(6)+BS(7)+BS(8)+BS(9)
      T=CBC(19)/ZM(19)
      FBR=(((A(1,M)/5.0)/.99045)/ABC(19))*1000.0
      WRITE(5,600)RNN,T,FBR
  600 FORMAT(//5X,'N.R.R.',F7.5,5X,'POPULATION INERTIA','=',F8.4,5X,'CBR
     1 FOR FEMALES=',F7.3)
C     CALCULATE %15,%15-45, %45+
  601 AY(1)=0.0
      DO 704 I=1,3
  704 AY(I+1)=AY(I)+A(I,M)
      BY=AY(4)/ABC(19)
      CY(4)=0.0
      DO 705 I=4,9
  705 CY(I+1)=CY(I)+A(I,M)
      DY=CY(10)/ABC(19)
      EY(10)=0.0
      DO 706 I=10,18
  706 EY(I+1)=EY(I)+A(I,M)
      GY=EY(19)/ABC(19)
      WRITE(5,707)BY,DY,GY
  707 FORMAT(75X,'%15=',F6.4,3X,'%15-44',F7.4,3X,'45+=',F6.4)
      AKL(1)=0.0
      AJL(1)=0.0
C     CALCULATE SLOPE OR DEVIATION
      DO 403 I=2,18
      AKL(I)=ABA(I-1)-ABA(I)
  403 AJL(I)=AJL(I-1)+AKL(I)
      AJL(19)=AJL(18)/17.0
      CKL(1)=0.0
```

```
C       STANDARD DEVIATION
        DO 404 I=2,18
        BKL(I)=(AKL(I)-AJL(19))*(AKL(I)-AJL(19))
  404   CKL(I)=CKL(I-1)+BKL(I)
C        VARIANCE
        CKL(19)=CKL(18)/17.0
        CKL(20)=SQRT(CKL(19))
C       COEFFICIENT OF VARIATION
        YKL=(CKL(20)/AJL(19))*100.0
        WRITE(6,405)(KK(I),I=1,9)
  405   FORMAT(//2X,'AGE GROUP',5X,9(4X,I2,4X))
        WRITE(6,406)(AKL(I),I=1,9)
  406   FORMAT(/4X,'% SLOPE',5X,9(2X,F6.3,2X))
        WRITE(6,405)(KK(I),I=10,18)
        WRITE(6,406)(AKL(I),I=10,18)
        WRITE(6,407)AJL(19),CKL(20),CKL(19),YKL
  407   FORMAT(//5X,'SLOPE MEAN',F10.4/5X,'SLOPE STD. DEV.',F10.4/5X,'SLOPE
       1E VARIANCE ',F10.4/5X,'COEFFICIENT OF VARIATION',F10.4)
  100   CONTINUE
        RETURN
        END
```

CHAPTER III

STABILIZATION PROCESS OF HUMAN POPULATIONS

In this chapter the stabilization process of a population is analysed. The chapter is divided into three sections. The first section gives general background of the process of stabilization. In the second section, a new method of looking at the stabilization process is proposed and mathematical proof, in terms of necessary and sufficient conditions of stability, is provided to justify the suggested approach. The third section deals with the analytical part of the stabilization process using the suggested approach.

GENERAL BACKGROUND OF THE PROCESS OF STABILIZATION

In the stable population theory fertility and mortality schedules are presumed to be constant, with fixed age structure. The birth rate (b) and the death rate (d) are calculated as

$$b = \int_{o}^{w} C(a)\ m(a)\ da$$

$$d = \int_{o}^{w} C(a)\ \mu(a)\ da$$

where C(a) is the proportion of females age a to a+da,

$m(a)$ is the annual rate of bearing female children and $\mu(a)$ is the age specific mortality rate (Coale, 1956).

There are two types of births involved in population projections or in the process of stabilization of a population. One type of births is to those females who are from the initial population and remain in the reproductive age during the projections and the other is those female births which are from the females born themselves in the projection period (Coale, 1972, p. 63). Then

$$B(t) = F(t) + \int_{0}^{\beta} B(t-a)\ p(a)\ m(a)\ da$$

where $F(t)$ are births to the females of initial population, $B(t)$ is total number of births at time t, $p(a)$ is proportion surviving from birth to age a, $m(a)$ is the annual rate of bearing female children at age a. When the females of initial population pass the reproductive age, then $F(t) = 0$ and the equation becomes:

$$B(t) = \int_{0}^{\beta} B(t-a)\ p(a)\ m(a)\ da$$

($t \geq \beta$ where β is the oldest age of non-zero fertility).

It is demonstrated that the birth sequence is the sum of an infinite number of exponentials and that the

real (non-oscillatory) root is larger than the real part of any complex (oscillatory) root. Thus the oscillatory terms become negligible relative to the non-oscillatory term (Coale, 1972, p. 64).

Although much work has been done on the stable population theory, yet least attention has been paid to the question of how a given population converges to its stable form (Bourgeois-Pichat, 1971; U.N., 1968; and Coale and Demeny, 1966). The well-known work on this topic is that of Coale (1968 and 1972). The convergence of a population toward stability has been given extensive treatment by Coale under four different modes of inquiry:

(1) How the sequence of births implies convergence;

(2) How specific characteristics of fertility and mortality regulate the components of exponential function;

(3) How the initial age-distribution interacts with the fertility schedules to determine the coefficients of exponential term; and

(4) How the approach to stability can be visualized.

Our major concern is with the last topic covered by Coale; that is, how the approach to stability is visualized. Our analysis is limited to the extent that (a) we do not provide mathematical relationships between different

characteristics of age-distributions. Instead, we present a description of the manners in which a population converges towards its stable form; and (b) only one mortality schedule and one initial age-distribution are used with various fertility schedules.

METHOD OF VISUALIZING STABILIZATION PROCESS

In this section, a minor new approach, that of the use of slope, which could be useful as an alternative in visualizing the stabilization process is presented.

It is well-documented in the literature that a higher fertility schedule would eventually produce a younger population and a lower fertility schedule an older one. In other words, for every fertility schedule there is a unique finite age-distribution which is independent of the initial age-distribution.

There are different ways of characterizing an age-distribution. The raw distribution of the proportions at each age is often used in the analysis of the age-distributions. In a closed population, the cohort of persons moves into successive age groups over a period of time. The magnitude (size) of the proportions is determined by fertility and mortality schedules. One of the ways to look at the changes in the size of proportions of an age-distribution is to calculate the difference between the proportion of two successive age-groups. This difference is

termed the slope. Since the slopes are a simple and direct consequence of the age-distribution, the notion of slopes may be taken as another way of characterizing a distribution.

It is important to note that every distribution, no matter what its nature, has some kind of slope(s). If we know the slope(s) of the distribution and at least one element of the age-distribution, the whole distribution can be reconstructed. However, the average slope must be recognized as the difference between the proportions of the first and the last age groups divided by the number of age groups minus one. In particular, for a discrete age-distribution, average slope is $(P_i - P_y)/(y-1)$, where P_i is the proportion in the ith age interval and y is the number of discrete age intervals. If we know the (y-1) slopes (henceforth called slope distribution) and the proportion of any one age-group to the total age-distribution, the entire age-distribution can be reconstructed. Thus, it is realistic to say that for every fertility schedule there is a finite age-distribution and every finite age-distribution has a slope distribution of its own.

NECESSARY AND SUFFICIENT CONDITION FOR STABILITY USING SLOPE NOTION

The existing literature suggests that when an age-distribution does not change or remains constant over a period of time the age-distribution is stable.

Keyfitz (1968, pp. 40-47) demonstrated that the dominant root of the M matrix (or the eigenvalue) λ_1, when constant for all age-groups, may be taken as a condition of stability. The constant value of λ_1 meets the requirements of sufficiency and necessity for population stability as it involves not only the age-structure but also the growth potential ($r = \log \lambda_1$).

Besides λ_1, we suggest that the notion of slope(s), as defined earlier in this chapter, can be used as a useful indicator of stability. Like λ_1, it can be mathematically demonstrated that the notion of slope(s) meets the requirements of necessary and sufficient conditions of stability.

The concepts such as age-distribution, slope distribution, stability and the difference between the slope distributions are used as discrete variables to develop the slope notion in deducing the necessary and sufficient conditions of population stability. These concepts are defined as:

Definition 1. Age-Distribution: The age distribution at time t is

$$\left\{p_x^t,\ 0 \le p_x^t \le 1,\ \sum_{x=1}^{y} p_x^t = 1,\ x = 1, 2, 3, \ldots\ldots, y\right\}$$

where p_x^t is the proportion of persons in the age-group x at time t and y is the number of age groups in an age-distribution.

Definition 2. Slope Distribution: The slope distribution of an age-distribution $\{p_x^t\}$ at time t is

$$\left\{d_x^t = \left\{p_x^t - p_{x+1}^t\right\},\ x = 1, 2, 3, \ldots\ldots, y-1\right\}$$

Definition 3. Population Stability: A population with the age-distribution given by definition 1 is said to have reached stability if there is a positive integer N such that for all n < N, $\sum_{x=1}^{y} |p_x^{t_n} - p_x^{t_{n-1}}| = 0$.

Definition 4. Difference between the Slope Distributions at Times t_1 and t_2. The difference between the slope distributions at times t_1 and t_2 is defined to be

$$\sum_{x=1}^{y-1} \left| d_x^{t_1} - d_x^{t_2} \right|$$

Theorem: A population has reached stability if and only if there is a positive integer N such that for all n > N the difference in the slope distributions at times t_n and t_{n-1} is zero.

Proof: Let $\left\{p_x^t\right\}$ be the age distribution of the population at time t. For each value of t (t = 1, 2, 3, . . . , N, ...n) in the population projections we get an age-distribution as a result of the interaction between fertility and mortality schedules. The age-distributions may be presented in the following tabular form.

Age Group	Projection Time t_0	t_1	t_2	t_N	t_n
1	$P_1^{t_0}$	$P_1^{t_1}$	$P_1^{t_2}$ -------	$P_1^{t_N}$ -------	$P_1^{t_n}$
2	$P_2^{t_0}$	$P_2^{t_1}$	$P_2^{t_2}$ -------	$P_2^{t_N}$ -------	$P_2^{t_n}$
3	$P_3^{t_0}$	$P_3^{t_1}$	$P_3^{t_2}$ -------	$P_3^{t_N}$ -------	$P_3^{t_n}$
4	$P_4^{t_0}$	$P_4^{t_1}$	$P_4^{t_2}$ -------	$P_4^{t_N}$ -------	$P_4^{t_n}$
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
y	$P_y^{t_0}$	$P_y^{t_1}$	$P_y^{t_2}$ -------	$P_y^{t_N}$ -------	$P_y^{t_n}$
TOTAL	1.0	1.0	1.0-------	1.0-------	1.0

By definition 3, if a population distribution is stable, there is a positive integer N such that for all $n > N$,

$$\sum_{x=1}^{y} \left| p_x^{t_n} - p_x^{t_{n-1}} \right| = 0.$$

Thus for each x $(x = 1, 2, 3, \ldots, y)$ and for all $n > N$

$$| p_x^t n - p_x^t n-1 | = 0 \text{ or } p_x^t n = p_x^t n-1$$

Therefore, for each x $(x= 1, 2, 3, \ldots, y-1)$ and for all $n > N$

$$d_x^t n - d_x^t n-1 = \left\{p_x^t n - p_{x+1}^t n\right\} - \left\{p_x^t n-1 - p_{x+1}^t n-1\right\}$$

$$= \left\{p_x^t n - p_x^t n-1\right\} - \left\{p_{x+1}^t n - p_{x+1}^t n-1\right\} = 0.$$

and hence for all $n > N$

$$\sum_{x=1}^{y-1} | d_x^t n - d_x^t n-1 | = 0.$$

On the other hand, if there is a positive integer N such that for all $n > N$

$$\sum_{x=1}^{y-1} | d_x^t n - d_x^t n-1 | = 0$$

then for each x $(x = 1, 2, 3, \ldots, y-1)$

$$| d_x^t n - d_x^t n-1 | = 0 \text{ or } d_x^t n = d_x^t n-1$$

which, by definition 2 is equivalent to

$$p_x^t n - p_{x+1}^t n = p_x^t n-1 - p_{x+1}^t n-1$$

or

$$p^t_x n - p^t_x n{-}1 = p^t_{x+1} n - p^t_{x+1} n{-}1$$

Thus

$$p^t_1 n - p^t_1 n{-}1 = p^t_2 n - p^t_2 n{-}1 = \ . \ . \ . = p^t_y n - p^t_y n{-}1 = k, \text{ say.}$$

Thus

$$\sum_{x=1}^{y} \left\{ p^t_x n - p^t_x n{-}1 \right\} = ky.$$

But by definition 1:

$$\sum_{x=1}^{y} p^t_x n = \sum_{x=1}^{y} p^t_x n{-}1 = 1, \text{ so } ky = 0 \text{ or}$$

$k = 0.$

Thus $p^t_x n - p^t_x n{-}1 = 0$, $(x = 1, 2, \ . \ . \ . \ , y)$

so that

$$\sum_{x=1}^{y} \left| p^t_x - p^t_x n{-}1 \right| = 0, \text{ and the distribution is}$$

stable.

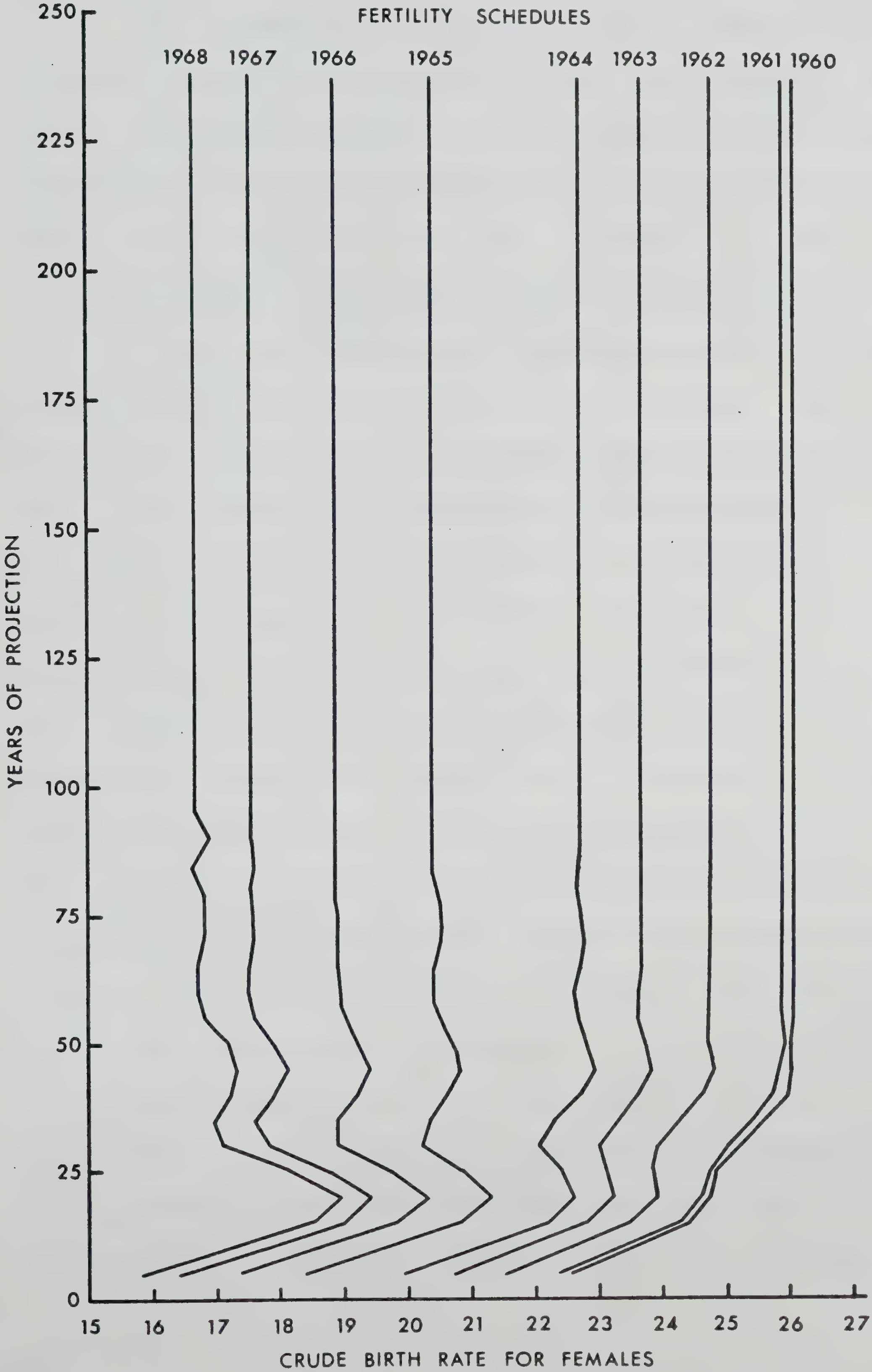
ANALYSIS AND DISCUSSION

Figure 3.1 shows how during the process of stabilization the crude birth rate for females for each of the fertility schedules behaves. It is clear from Figure 3.1 that there are significant changes in the crude birth rate for females during the early period of the process of stabilization for each fertility schedule. After about 70 to 80 years of projection, the birth rate for each fertility schedule remains approximately, if not completely, constant. It is not surprising that the changes in the female birth rate during the first few decades of population projection show more or less the same trend for all fertility schedules, given that all of them come from a narrow choice of alternatives.

We know that the different fertility schedules would produce different stable age-distributions even though the mortality schedule and the initial age-distribution remain the same for all fertility schedules. During the examination of Figure 3.1, one must ask why there are irregularities or oscillations in the female birth rate during the early part of the population projections, what causes the oscillations to be significant or insignificant, and why these oscillations disappear with the passage of time during the process of stabilization

FIGURE 3.1

Crude Birth Rate for Females during the Process of Convergence to Stable Form for U.S.A. Fertility Schedules 1960-1968 by Years of Projection



of a population?

The questions raised above may be answered in the light of slopes or difference between the successive age-groups of a proportionate age-distribution. The slopes between the successive age-groups of an age-distribution form a distribution of their own. Tables 3.1, 3.2, and 3.3 are constructed to show the arithmetic mean and the variance of the slope distributions for different points of time in population projections by fertility schedules. The arithmetic mean of the slope distribution is calculated as the sum of the slopes in an age-distribution divided by (y-1) age-groups. Variance is the measure of dispersion and is equal to the square of the standard deviation of the slope distribution. The variance of the slope distribution is the distribution around the mean slope. The slope is the difference between the proportions of successive age-groups in an age-distribution. The age-distribution has fixed fertility and mortality schedules throughout the period of population projections and/or during the process of stabilization. Tables 3.1, 3.2, and 3.3 demonstrate that for quite some time in the projections the mean of the slope distribution does not show any consistent pattern. In the early part of the projections, the mean of the slope distribution goes on increasing and then declines, again increases and declines, and so on. On the other hand, variance

TABLE 3.1

Mean and Variance of the Slope Distribution During the Process of Stabilization

by Fertility Schedules and by Time of Projections

Time of Projection (in years)	FERTILITY SCHEDULES 1960		1961		1962	
	Mean	Variance	Mean	Variance	Mean	Variance
5	0.6111	0.2349	0.6067	0.2283	0.5806	0.2045
10	0.6345	0.2252	0.6304	0.2197	0.6062	0.2014
15	0.6575	0.2194	0.6538	0.2152	0.6318	0.2010
20	0.6697	0.1884	0.6662	0.1850	0.6448	0.1729
25	0.6747	0.1542	0.6704	0.1504	0.6440	0.1364
30	0.6843	0.1312	0.6789	0.1262	0.6464	0.1068
35	0.6964	0.1196	0.6906	0.1142	0.6560	0.0917
40	0.7072	0.1085	0.7015	0.1034	0.6676	0.0810
45	0.7117	0.0999	0.7061	0.0952	0.6723	0.0734
50	0.7165	0.0929	0.7108	0.0882	0.6759	0.0664
55	0.7243	0.0845	0.7183	0.0796	0.6821	0.0571
60	0.7301	0.0777	0.7240	0.0727	0.6873	0.0492
65	0.7301	0.0755	0.7240	0.0706	0.6874	0.0463
70	0.7271	0.0756	0.7211	0.0708	0.6843	0.0462
75	0.7255	0 0754	0.7194	0.0705	0.6822	0.0455
80	0.7254	0.0765	0.7191	0.0717	0.6813	0.0468
85	0.7282	0.0737	0.7220	0.0690	0.6840	0.0451
90	0.7278	0.0740	0.7217	0.0691	0.6851	0.0438
95	0.7272	0.0741	0.7212	0.0692	0.6843	0.0437
100	0.7266	0.0745	0.7205	0.0696	0.6832	0.0442
105	0.7267	0.0745	0.7205	0.0697	0.6830	0.0446

TABLE 3.1 (Continued)

Time of Projection (in years)	FERTILITY SCHEDULES 1960		1961		1962	
	Mean	Variance	Mean	Variance	Mean	Variance
110	0.7272	0.0742	0.7210	0.0694	0.6836	0.0443
115	0.7273	0.0741	0.7212	0.0692	0.6841	0.0440
120	0.7271	0.0742	0.7210	0.0693	0.6839	0.0439
125	0.7270	0.0743	0.7208	0.0694	0.6836	0.0441
130	0.7270	0.0743	0.7208	0.0694	0.6835	0.0442
135	0.7271	0.0743	0.7209	0.0694	0.6836	0.0442
140	0.7271	0.0742	0.7210	0.0693	0.6838	0.0441
145	0.7271	0.0742	0.7210	0.0693	0.6838	0.0440
150	0.7270	0.0743	0.7209	0.0694	0.6837	0.0441
155	0.7270	0.0743	0.7209	0.0694	0.6836	0.0441
160	0.7271	0.0743	0.7209	0.0694	0.6837	0.0441
165	0.7271	0.0742	0.7209	0.0694	0.6837	0.0441
170	0.7271	0.0742	0.7209	0.0694	0.6837	0.0441
175	0.7271	0.0743	0.7209	0.0694	0.6837	0.0441
180	0.7271	0.0743			0.6837	0.0441
185	0.7271	0.0743			0.6837	0.0441
190	0.7271	0.0743			0.6837	0.0441
195	0.7271	0.0743			0.6837	0.0441
200					0.6837	0.0441
205					0.6837	0.0441
210						
215						

TABLE 3.2

Mean and Variance of the Slope Distributions During the Process of Stabilization by Fertility Schedules and by Time of Projections

Time of Projection (in years)	FERTILITY SCHEDULES					
	1963		1964		1965	
	Mean	Variance	Mean	Variance	Mean	Variance
5	0.5571	0.2060	0.5357	0.2262	0.4898	0.3299
10	0.5838	0.2036	0.5631	0.2220	0.5186	0.3164
15	0.6111	0.2039	0.5918	0.2205	0.5491	0.3046
20	0.6249	0.1758	0.6062	0.1908	0.5639	0.2659
25	0.6203	0.1377	0.5985	0.1512	0.5500	0.2228
30	0.6170	0.1044	0.5902	0.1155	0.5323	0.1850
35	0.6241	0.0850	0.5947	0.0916	0.5311	0.1514
40	0.6358	0.0723	0.6062	0.0755	0.5409	0.1244
45	0.6409	0.0641	0.6117	0.0655	0.5459	0.1068
50	0.6438	0.0569	0.6140	0.0577	0.5473	0.0955
55	0.6488	0.0468	0.6180	0.0467	0.5496	0.0819
60	0.6533	0.0371	0.6217	0.0349	0.5517	0.0635
65	0.6532	0.0324	0.6212	0.0272	0.5498	0.0455
70	0.6500	0.0308	0.6178	0.0233	0.5452	0.0312
75	0.6475	0.0293	0.6150	0.0201	0.5414	0.0199
80	0.6460	0.0305	0.6130	0.0207	0.5381	0.0165
85	0.6485	0.0299	0.6152	0.0214	0.5394	0.0214
90	0.6511	0.0270	0.6193	0.0169	0.5483	0.0129
95	0.6499	0.0265	0.6179	0.0158	0.5456	0.0091
100	0.6485	0.0269	0.6160	0.0159	0.5426	0.0076
105	0.6481	0.0276	0.6153	0.0169	0.5412	0.0090

TABLE 3.2 (Continued)

Time of Projection (in years)	FERTILITY SCHEDULES 1963 Mean	1963 Variance	1964 Mean	1964 Variance	1965 Mean	1965 Variance
110	0.6488	0.0275	0.6161	0.0171	0.5424	0.0104
115	0.6494	0.0270	0.6170	0.0165	0.5440	0.0099
120	0.6494	0.0268	0.6170	0.0160	0.5441	0.0089
125	0.6489	0.0269	0.6165	0.0160	0.5433	0.0083
130	0.6487	0.0271	0.6162	0.0163	0.5428	0.0086
135	0.6489	0.0271	0.6163	0.0165	0.5429	0.0091
140	0.6491	0.0270	0.6166	0.0164	0.5433	0.0092
145	0.6491	0.0269	0.6167	0.0162	0.5435	0.0089
150	0.6490	0.0269	0.6165	0.0162	0.5433	0.0087
155	0.6489	0.0270	0.6164	0.0162	0.5431	0.0087
160	0.6489	0.0270	0.6164	0.0163	0.5431	0.0089
165	0.6490	0.0270	0.6165	0.0163	0.5433	0.0089
170	0.6490	0.0270	0.6165	0.0163	0.5433	0.0089
175	0.6490	0.0270	0.6165	0.0163	0.5433	0.0088
180	0.6490	0.0270	0.6165	0.0163	0.5432	0.0088
185	0.6490	0.0270	0.6165	0.0163	0.5432	0.0089
190	0.6490	0.0270			0.5432	0.0089
195	0.6490	0.0270				
200	0.6490	0.0270				
205	0.6490	0.0270				
210	0.6490	0.0270				
215						

TABLE 3.3

Mean and Variance of the Slope Distributions During the Process of Stabilization

by Fertility Schedules and by Time of Projections

Time of Projection (in years)	FERTILITY SCHEDULES					
	1966		1967		1968	
	Mean	Variance	Mean	Variance	Mean	Variance
5	0.4595	0.4439	0.4318	0.5793	0.4157	0.6723
10	0.4894	0.4221	0.4624	0.5489	0.4468	0.6370
15	0.5205	0.3999	0.4944	0.5168	0.4792	0.5990
20	0.5345	0.3522	0.5081	0.4602	0.4927	0.5371
25	0.5158	0.3073	0.4857	0.4138	0.4683	0.4900
30	0.4926	0.2701	0.4570	0.3789	0.4360	0.4579
35	0.4883	0.2305	0.4492	0.3349	0.4260	0.4121
40	0.4968	0.1951	0.4565	0.2920	0.4329	0.3653
45	0.5004	0.1704	0.4594	0.2605	0.4355	0.3299
50	0.5004	0.1557	0.4583	0.2420	0.4336	0.3092
55	0.5018	0.1394	0.4584	0.2226	0.4328	0.2882
60	0.5032	0.1147	0.4588	0.1907	0.4324	0.2514
65	0.5003	0.0860	0.4547	0.1489	0.4277	0.2003
70	0.4941	0.0597	0.4472	0.1069	0.4194	0.1464
75	0.4890	0.0377	0.4407	0.0696	0.4119	0.0970
80	0.4845	0.0276	0.4348	0.0484	0.4048	0.0662
85	0.4856	0.0361	0.4350	0.0607	0.4044	0.0808
90	0.4986	0.0245	0.4532	0.0458	0.4263	0.0635
95	0.4947	0.0177	0.4480	0.0348	0.4203	0.0491
100	0.4906	0.0139	0.4426	0.0274	0.4139	0.0387
105	0.4886	0.0154	0.4398	0.0286	0.4105	0.0394

TABLE 3. 3 (Continued)

Time of Projection (in years)	FERTILITY SCHEDULES 1966 Mean	1966 Variance	1967 Mean	1967 Variance	1968 Mean	1968 Variance
110	0.4902	0.0180	0.4418	0.0329	0.4127	0.0451
115	0.4924	0.0179	0.4448	0.0336	0.4163	0.0466
120	0.4926	0.0164	0.4452	0.0315	0.4169	0.0439
125	0.4915	0.0153	0.4437	0.0294	0.4138	0.0406
130	0.4907	0.0155	0.4426	0.0293	0.4140	0.0421
135	0.4909	0.0162	0.4428	0.0305	0.4149	0.0431
140	0.4915	0.0165	0.4436	0.0311	0.4154	0.0428
145	0.4917	0.0162	0.4439	0.0308	0.4151	0.0420
150	0.4915	0.0159	0.4437	0.0302	0.4146	0.0417
155	0.4913	0.0158	0.4433	0.0301	0.4145	0.0420
160	0.4912	0.0160	0.4433	0.0303	0.4148	0.0423
165	0.4914	0.0161	0.4434	0.0305	0.4149	0.0424
170	0.4914	0.0161	0.4436	0.0305	0.4149	0.0422
175	0.4914	0.0160	0.4436	0.0304	0.4148	0.0420
180	0.4914	0.0160	0.4435	0.0303	0.4147	0.0421
185	0.4913	0.0160	0.4434	0.0303	0.4148	0.0422
190	0.4914	0.0160	0.4435	0.0304	0.4148	0.0422
195	0.4914	0.0160	0.4435	0.0304	0.4148	0.0422
200	0.4914	0.0160	0.4435	0.0304	0.4148	0.0422
205			0.4435	0.0304		
210			0.4435	0.0304		
215			0.4435	0.0304		

decreases monotonically for almost the first 75 years of the projections. After this period, both the mean and variance of the slope distributions fluctuate in their respective values and continue to do so until they stabilize along with the age-distributions. Note that the convergence of variance to stability shows higher velocity in the first six or seven decades of projections than the convergence of variance towards stability after the first six or seven decades. The fluctuations in the values of mean and variance after 75 years of projections are not, however, as significant as the fluctuations in the values of mean during the early period of projections.

A thorough examination of each computer output of projections reveals that at the beginning of the projection period some of the slopes are negative. As the time of projections is prolonged, the negative signs of the slopes disappear. Obviously, the negative slopes which disappear during the process of stabilization of a population are inherited from the initial age-distribution (U.S.A. 1963 distribution) because of the selectivity of migration by age which sets a hump during an early part of projections (Keyfitz and Flieger, 1968, p. 13). Other reasons for negative slopes may be age-misreporting and age-heaping, age-selective over- or under-enumeration, etc..

However, if some slopes remain negative throughout the process of stabilization, the fertility must be below the replacement level.

One may argue that, during the stabilization process of a population, the immediate effects of interaction between the vital processes are to get rid of the disproportions in the age-groups of the initial population. In the early period of population projections, the disproportionate age-groups move along as the successive age-groups. A closed population achieves a smooth distribution only when the persons in the disproportionate age-group(s) die out in the course of projections. We have observed that in all fertility schedules it takes almost 85 years for the U.S.A. 1963 age-distribution to achieve smoothness in the age distribution. In fact, all negative signs for the slopes, if any, disappear in relatively higher fertility schedules (1960, 1961, 1962, and 1963) after 35 years of projections; in medium fertility schedules (1964 and 1965) after almost 50 years; and in low fertility schedules (1966, 1967, and 1968) after about 70 years. It is also noted that soon after the disappearance of negative signs in the slopes smoothness in the distribution is not possible, since the slope for the respective age-groups is close to zero and it takes another 10 to 15 years to achieve slopes of reasonable size. The effects stemming from the initial age-distribution, in this

context, disappear in 85 years, which, incidentally, is the upper limit of the maximum age of the persons in the initial population.

To elaborate the above point, we have selected three fertility schedules of 1960, 1965, and 1968 as high, medium, and low fertility schedules, respectively, to present the stabilization process in a graphic form. There are three selected slope distributions for each of the selected fertility schedules. Slopes for age 85+ must be ignored as the interval for this age-group is open and is not consistent with that of other age-groups. For each of the selected fertility schedules, three points of time during the stabilization process are selected to demonstrate the process. These points are: (a) slope distributions after 50 years of projection, (b) slope distributions after 85 years of projection, and (c) slope distribution of the stable population. We have also presented the slope distribution of the initial population in Figures 3.2, 3.3, and 3.4 to highlight the changes in the initial age-distribution during the process of stabilization.

It is evident from Figures 3.2, 3.3, and 3.4 that after 50 years of population projections the slope distributions have larger oscillations than those after 85 years of projections, while the slope distributions of the stable population show only such oscillations as permitted by the

FIGURE 3.2

Slope Distribution of Population during the Process of Convergence to Stable Form for U.S.A., 1960 Fertility Schedule

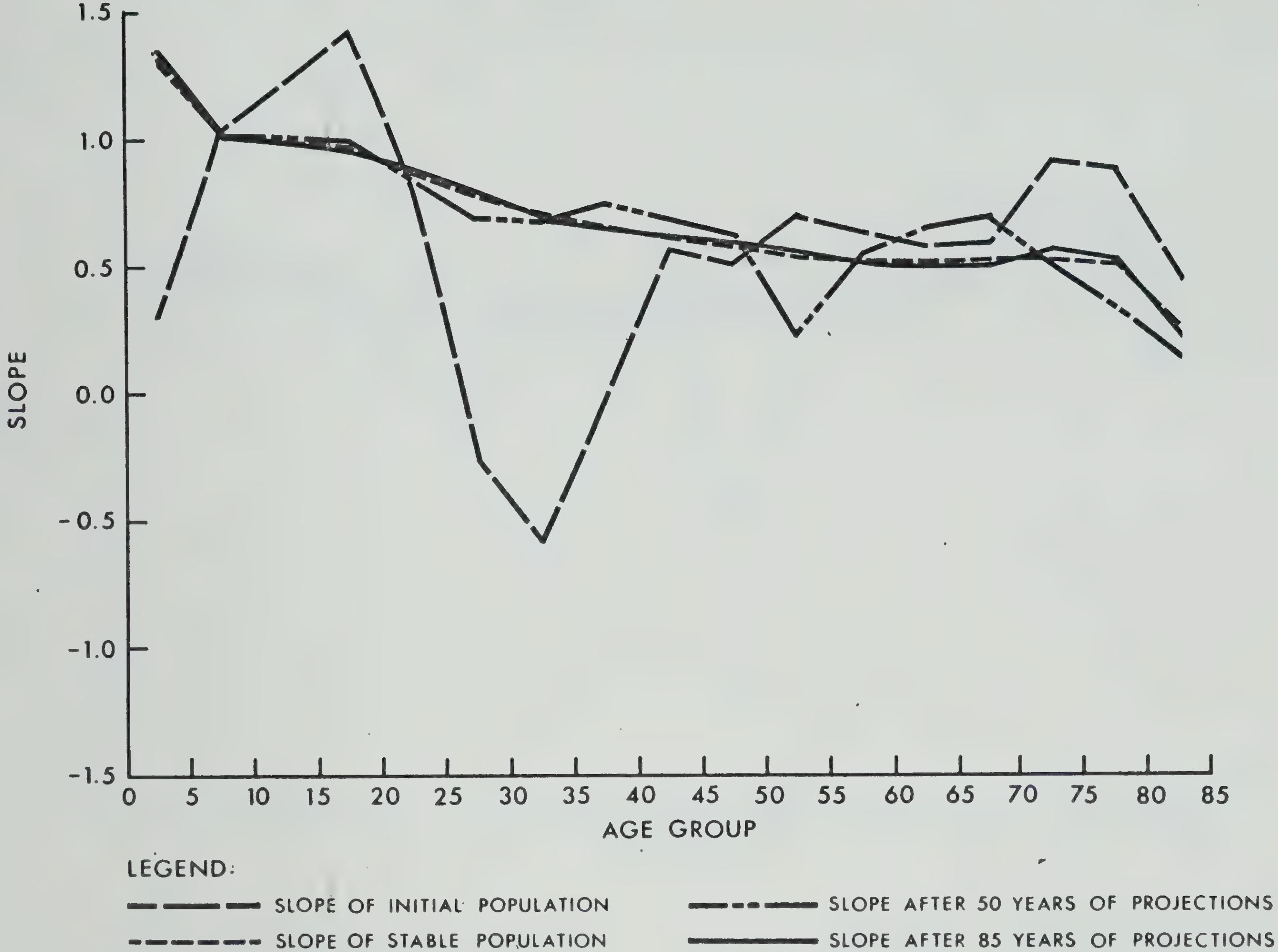


FIGURE 3.3
Slope Distribution of Population during the Process of Convergence to Stable Form for U.S.A., 1965 Fertility Schedule

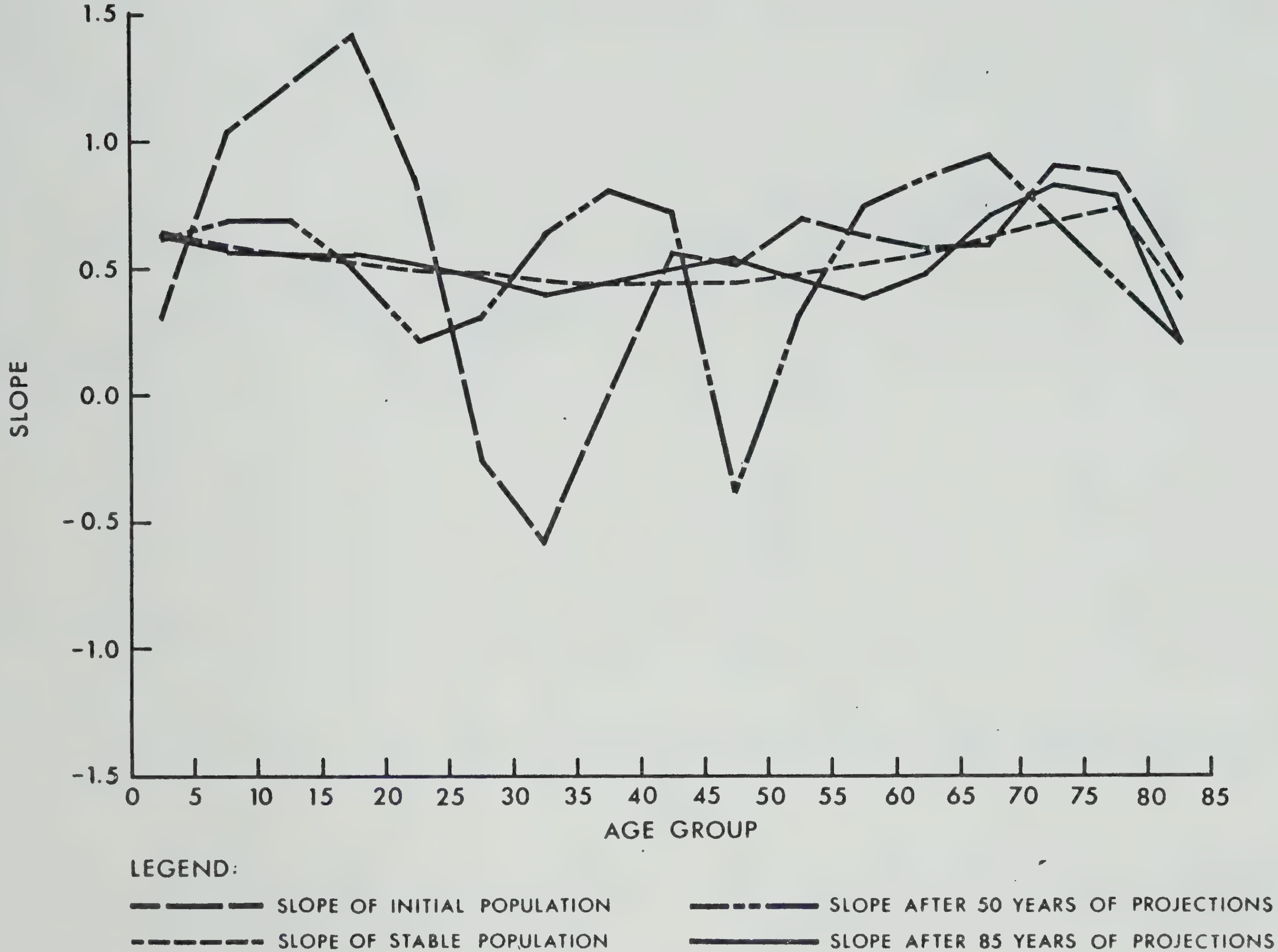
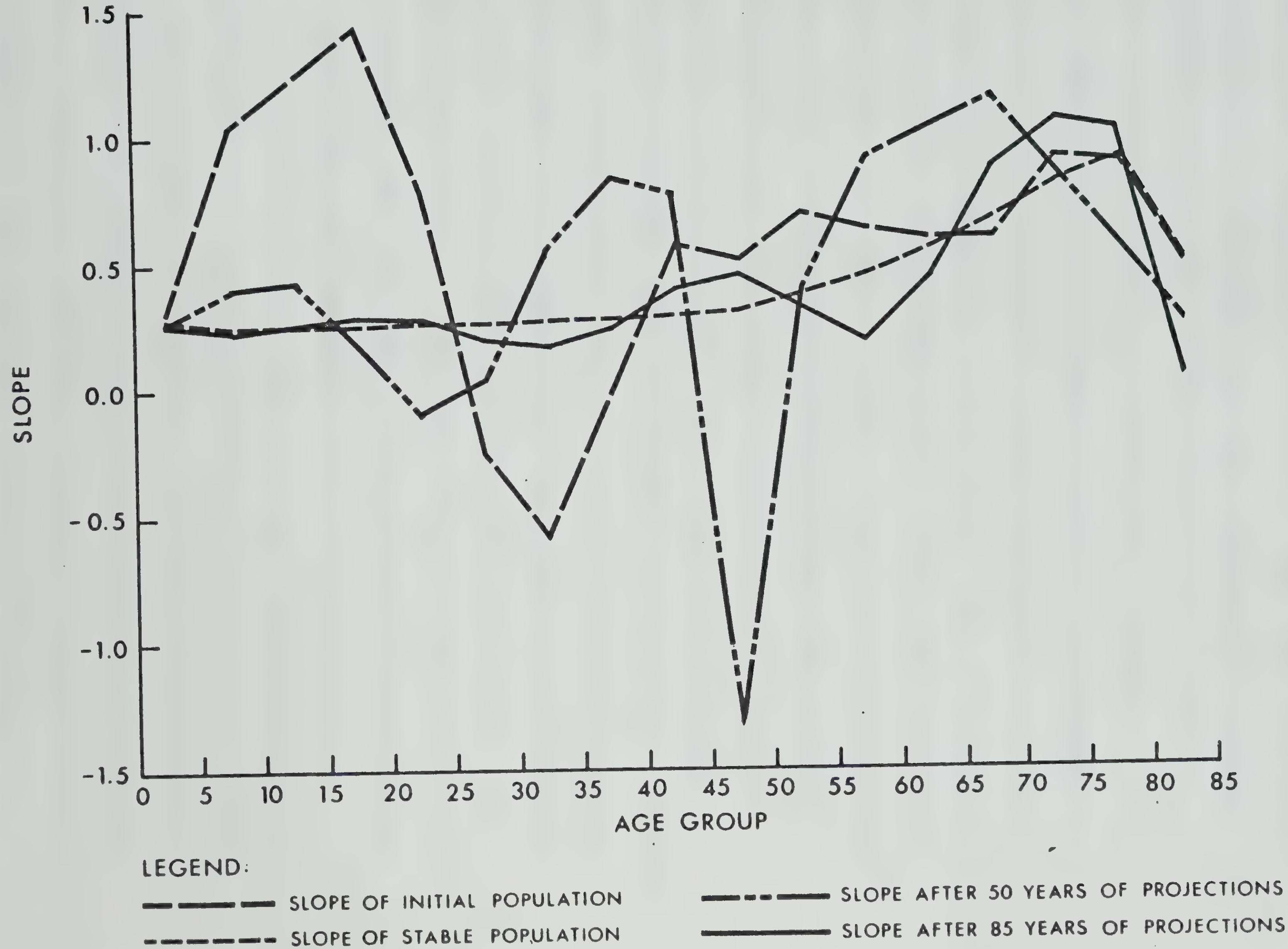


FIGURE 3.4
Slope Distribution of Population during the Process of Convergence to Stable Form for U.S.A., 1968 Fertility Schedule



definition of stability. It is also clear that the slope distribution for the high fertility schedule has smaller oscillations that that of the medium and low fertility schedules after 50 years of projections. Similarly, the slope distribution of the medium fertility has smaller oscillations than oscillations in the low fertility slope distribution (see Figures 3.2 and 3.3). After 85 years of projections, the slope distribution of the high fertility schedule (1960) is close to that of the stable distribution (Figure 3.2). As fertility declines from 1960 to 1968 monotonically, the slope distributions after 85 years of projections for the 1965 and 1968 fertility schedules are distinctly different from the respective stable slope distributions. From Figures 3.2, 3.3, and 3.4, one may conclude that high fertility schedules produce smaller oscillations than low fertility schedules during the early part of the stabilization process. The examination of the computer program suggests that the negative slopes vanish more quickly with a high fertility schedule than with a low one.

So far, we have seen that the high fertility schedules diminish the negative signs in the slopes earlier than the low fertility schedules. To show how fertility schedules cause changes in the age-structure over a period of time, Tables 3.1, 3.2, and 3.3 provide some guidelines. From Tables 3.1, 3.2, and 3.3, one can safely state that a high fertility schedule would produce slopes of larger

size than the low fertility schedule. The size of the slopes is reflected in the size of the arithmetic mean of a slope distribution.

It is observed that when higher fertility schedules produce large slopes in the distributions the variance of the slope distributions do not show a linear relationship with the fertility levels. In the early years of projection, the lower the fertility level, the higher the variance of the slope distribution. This is because the low fertility schedules cannot effectively smooth the distribution. By the time the stabilization process is complete, the relationship between the fertility level and the variance has no consistent pattern. For example, fertility declines from 1960 to 1968 monotonically and so does the mean of the slope distribution, but the variance of the slope distribution declines from 1960 to 1965 and increases from 1966 to 1968.

We know that the initial age-distribution does not have any effect on the stable age-distribution and also that there is only one mortality schedule used for all fertility schedules. Therefore, the variance of the slope distribution, in our analysis, is affected by the fertility factor only. The inconsistent pattern in the variance of the slope distributions at the time of stability is due to the changes in the age of the stable distributions. The aging process of a population is extensively treated

in Chapter IV. However, it may be stated here that with the fertility schedules 1960 to 1965, the corresponding stable age distributions change from "young" age to "moderate" age. The variance of the slope distribution decreases for 1960 to 1965 because of the shift in the proportions toward the middle age groups. On the other hand, from 1965 to 1968, the fertility schedules change the distributions from "moderate" to "older" age; that is, there is a shift in the proportions from middle to older age groups. Since the variance is calculated around the mean, the shift in the proportions in either direction of the middle age groups results in large variance. It may further be elaborated by assuming 1965 distribution as the "moderate" one and by looking at the variance from 1960 to 1965 and from 1965 to 1968. The age of the stable populations is discussed in detail in the discussion of the "index of dissimilarity" in Chapter IV.

STAGES OF THE STABILIZATION PROCESS

So far, we have highlighted some of the analytical aspects of the age-distributions and/or slope distributions during the process of stabilization. The data analysis and the related discussion in the preceding section deals with the effects of narrowly changing fertility. Such an analysis (by means of projection matrix approach) provides an opportunity to learn a great deal about the various stages of the stabilization process.

In our exercise, the stabilization process may be broadly studied in its initial, intermediate, and final stages.

Initial Stage: First of all, one has to consider in the initial population the component beyond age β and the segment under age β, where β is the highest age of childbearing. The population beyond the age of reproduction has no effect on the birth sequence, and if there are differences between the older population and the stable, these differences must disappear as the older population dies out in a maximum of $w-\beta$ years, where w is the highest age attained.

The effect of the population under age β is to determine, in conjunction with the net fertility function, the time sequence of births from the given moment on. At the start of the process of stabilization, the birth rate and the mean of the slope distribution fluctuate considerably. These fluctuations are mainly due to the characteristics of the initial age-distribution. The bumps and hollows around the long-run exponential birth sequence are significant and some of the slopes are negative. At this stage, the variations due to fertility are not easily detectable.

Intermediate Stage: As the persons from the initial population die out, that is, after the number of years of projection exceed the highest age attained (in our exercise,

85 years), the fluctuations in the mean of the slope distribution, birth rate, and other related measures become less and less significant. The negative signs of the slopes disappear. At this stage, effects of fertility are visible; that is, high fertility schedules tend to produce low oscillations while low fertility schedules tend to produce high oscillations. The mean of the slope distribution fluctuates but at low rate and all the slopes have positive signs.

Final Stage: In the final stage of the process of stabilization, the effects of the initial age-distribution are completely eliminated. The oscillations start disappearing, depending upon the level of fertility. With high fertility schedules, the oscillations disappear more rapidly that with low fertility schedules. The mean of the slope distribution is almost the same as that of the stable distribution. Other measures, such as birth rate, λ_1, and the slope variance stabilize. As the distribution achieves stability, meaningful relationships between different parameters are established.

CONCLUSION

The purpose of the present chapter is to illustrate the effects of various narrowly changing fertility schedules on one initial age-distribution when mortality is fixed. The process of stabilization is viewed

in the light of the changes in the slopes between the two successive age-groups of an age-distribution. A high fertility schedule with our given initial age-distribution and mortality level overcomes the oscillations more quickly than the low fertility schedule. There is a positive relationship between the mean of the slope and the level of fertility. The variance of the slope distribution is found to be an indicator of the aging of the distribution.

CHAPTER IV

FERTILITY AND THE AGING PROCESS OF HUMAN POPULATIONS

In the preceding chapter, we have discussed the effects of relatively narrow changes in fertility on the age-distribution of our selective population during the process of stabilization by means of the notion of slopes between the age-groups. This chapter consists of two parts: the first part deals with the differences in the nine fertility schedules to be used in our illustration, and the second explores the differences between the one initial and the nine stable age-distributions. The mortality level is constant for all nine fertility schedules (U.S.A., 1960-1968). In other words, with exactly the same initial age-distribution and the mortality schedule, the impact of different fertility schedules on the form of the respective stable age-distributions is analysed.

METHODOLOGY

Both simple and complex methods of comparing two or more distributions have been developed by social scientists. In demographic literature, standardization techniques by direct and indirect methods are commonly employed in comparing the rates of vital events (Jaffe, 1951, pp. 43-52). When two or more populations are compared, the simplest

method is to compute the proportionate distribution of each population. After the proportionate distributions for all populations are computed. the percentage point differences for each category in the distributions are calculated. The percentage point differences show how much one distribution differs from the other. For each category in the age-distribution (for example, an age-group is a category) the percentage point difference indicates the relative size of one distribution over the other with respect to that category.

The percentage point difference approach has a serious drawback. Whereas it does indicate the differences in the size of a category or categories, it does not show the relative change. If the percentage points for a category are large, the percentage point difference may also be large in size, signifying a major change. On the other hand, the difference between two small-sized percentages is more likely to be small in size which gives the impression of little change. In such situations the mere sizes of the percentage points for the categories in the distributions may enlarge or undermine the change (Bogue, 1969. pp. 117-121). To overcome this shortcoming, refinement can be made in the percentage point difference method by computing the 'index of relative composition' (Bogue, 1969; and Smith. 1966). The logic behind constructing the index of relative composition is that the absolute size of the percentage point differences is related to the

percentages from which they have been derived. To make compositional differences more meaningful, proportions of the percentage point difference to the respective base percentage are computed by the formula:

$$\text{Index of relative composition for category X} = \frac{\text{Proportion for X in distribution A - proportion for X in distribution B}}{\text{Proportion for X in distribution A}} \times 100$$

Based on the proportionate distributions, percentage point differences and the index of relative composition, other simple indices are constructed. These indices are used to measure 'unevenness,' 'concentration,' or the 'dissimilarities' between two or more distributions. The most commonly used index derived from proportionate distributions is known as 'index of population concentration' (Duncan, 1957), or 'change ratio' or 'index of dissimilarity' (Δ).

The index of dissimilarity has been used by a number of social scientists (Bogue, 1969; Keyfitz, 1968, and Mason, 1969). Keyfitz (1968, p. 47) uses the index of dissimilarity to show how different an initial distribution of a population is from the stable distribution. The index of dissimilarity is calculated by taking two percentage distributions, subtracting one distribution from the other to get a distribution of percentage point differences. These percentage point differences have positive (+) and (-)

negative signs. The sum of + signs always equals the sum of - signs. The sum of the categories of alike signs is called the index of dissimilarity. One can also take the total of all percentage point differences irrespective of the signs and divide the total by 2 to get the index of dissimilarity. Arithmetically:

$$\Delta = \frac{\sum_{X=1}^{n} \left| A_x - B_x \right|}{2}$$

where A and B are two proportionate distributions and X is a category.

It is clear from the formula that the index of dissimilarity shows the total change from one distribution to the other. It does not specify the pattern by which the two distributions differ. In other words, Δ does not give a clue to the compositional difference. In our analysis, we have made use of both the distribution of percentage point differences and the index of dissimilarity.

CHANGES IN THE U.S.A. FERTILITY 1960-1968

From Table 2.1 (Chapter II) we know that the fertility of the U.S.A. declined each year from 1960 to 1968. Table 4.1 shows the percentage distribution of age-specific fertility rates for each year. Table 4.1 is constructed by converting the age-specific fertility rates into the percentage of total fertility so that each fertility schedule adds up to 100.0.

TABLE 4.1

Percentage Distribution of Age-Specific Fertility Rates,

U. S. A., 1960 - 1968

Age-Group	1960	1961	1962	1963	1964	1965	1966	1967	1968
10 - 14	0.109	0.124	0.115	0.135	0.140	0.137	0.164	0.175	0.202
15 - 19	12.194	12.123	11.688	11.456	11.348	12.024	12.900	13.197	13.340
20 - 24	35.322	34.950	35.080	34.668	34.279	33.612	33.967	33.819	33.784
25 - 29	27.015	27.263	27.595	27.860	27.966	27.754	27.298	27.716	28.315
30 - 34	15.424	15.608	15.676	15.924	16.196	16.225	15.695	15.413	15.116
35 - 39	7.691	7.659	7.586	7.692	7.794	7.925	7.711	7.483	7.185
40 - 44	2.121	2.149	2.130	2.129	2.151	2.186	2.138	2.060	1.937
45 - 49	0.123	0.124	0.130	0.135	0.125	0.137	0.128	0.136	0.121
Total	100%	100%	100%	100%	100%	100%	100%	100%	100%

Table 4.2 shows the percentage point differences between the 1960 fertility schedule and each of the eight 1961 to 1968 fertility schedules. In other words, Table 4.2 shows how the age-specific fertility pattern of U.S.A. females between 1961 and 1968 differed from the 1960 pattern. Based on Table 4.2, Table 4.3 is constructed to show the index of relative composition of age-specific fertility rates from 1961 to 1968 taking the 1960 fertility schedule as the base. In fact, Tables 4.2 and 4.3 demonstrate, respectively, the absolute and relative changes in the fertility schedules of 1961 to 1968 in comparison with the 1960 fertility schedule. The negative signs in Tables 4.2 and 4.3 indicate for the respective age-groups proportionate increases from the 1960 fertility schedule and plus signs indicate decreases.

From Tables 4.2 and 4.3, one can see the compositional changes in the fertility schedules of 1961 to 1968 from that of 1960. The percentage contribution of age-group 10-14 increased for 1961 to 1968 as compared to the contribution of age-group 10-14 in 1960. Since these percentage points are very small in size, the magnitude of the absolute change demonstrated by Table 4.2 is small (or less than noticeable). The percentage contribution of age-group 15-19 declined monotonically for 1961, 1962, 1963, and 1964. In 1965, the contribution by age-group 15-19 toward the total fertility declined only slightly from that of 1960, and reversed the trend for 1966, 1967, and 1968, when the

[illegible]

TABLE 4. 2

Index of Dissimilarity and Percentage Point Difference Distributions for U. S. A., Fertility Schedules 1961 - 1968, Using 1960 Fertility Schedules as the Base

Fertility Schedules	AGE GROUPS 10-14	15-19	20-24	25-29	30-34	35-39	40-44	45-49	Index of Dissimilarity Δ
1961	-0.014	0.017	0.373	-0.248	-0.185	0.032	-0.028	-0.001	0.476
1962	-0.006	0.505	0.242	-0.579	-0.252	0.105	-0.009	-0.006	0.852
1963	-0.025	0.738	0.654	-0.845	-0.501	-0.001	-0.008	-0.012	1.392
1964	-0.031	0.845	1.043	-0.951	-0.773	-0.103	-0.030	-0.002	1.888
1965	-0.027	0.170	1.710	-0.739	-0.802	-0.234	-0.065	-0.013	1.880
1966	-0.055	-0.706	1.356	-0.282	-0.272	-0.019	-0.017	-0.005	1.356
1967	-0.065	-1.003	1.503	-0.701	0.011	0.208	0.061	-0.013	1.783
1968	-0.092	-1.146	1.538	-1.300	0.308	0.507	0.184	0.002	2.538

TABLE 4. 3

Index of Relative Composition of Age-Specific Fertility Rates for U. S. A. Fertility Schedules, 1961 - 1968 by Age, Using 1960 Fertility Schedule as the Base

Fertility Schedule	AGE GROUPS 10-14	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1961	-12.84	0.58	1.05	-0.91	-1.19	4.16	1.32	0.81
1962	- 5.50	4.14	0.68	-2.14	-1.63	1.36	0.42	4.87
1963	-22.93	6.05	1.85	-3.12	-3.24	0.01	0.37	9.75
1964	-28.44	6.92	2.95	-3.52	-5.01	1.34	1.41	1.62
1965	-24.77	1.39	4.84	-2.73	-5.19	3.05	3.60	10.56
1966	-50.45	-5.78	3.83	-1.04	-1.76	0.24	0.80	4.06
1967	-59.63	-8.22	4.25	-2.59	0.07	2.71	2.87	10.56
1968	-84.40	-9.39	4.35	-4.81	1.99	6.61	8.67	1.62

contributions by age-group 15-19 increased relative to that of the 1960 schedule.

The percentage contribution of age-group 20-24 toward the total fertility is lower every year in the 1961-1968 period than in 1960 (see Table 4.2). The minimum decline for this age-group is for 1962 while the maximum decline is for 1965. With these two exceptions, the decrease in the contribution of age-group 20-24 is monotonic over the period. Age-group 25-29 shows an altogether opposite trend to that of age-group 20-24. However, in spite of the decline in the overall contributions from 1961 to 1968, age-group 20-24 remains the modal age-group in all schedules. From 1961 to 1968, the age-group 20-24 always made greater contributions toward the total fertility than the contributions of the age-group 25-29. The contribution of the age-group 25-29 toward the total fertility increases monotonically from 1961 to 1964; for 1965 and 1966 the increment in the contribution decreases; and for 1967-1968, the contributions show an upward trend.

For the age-group 30-34 there is a monotonic increase in the percentage point contributions toward the total fertility for the years 1961 through 1965. In 1966 the trend in the percentage point contributions is reversed and for the last two years the contribution decreased relative to that of 1960. For age-group 35-39 the percentage contribution was lower in the years 1961, 1962, 1967 and 1968.

For the years 1963, 1964, 1965, and 1966 age-group 35-39 contributed proportionately more toward total fertility than in other years. The age-group 40-44 shows increase in the percentage point contributions for 1961 to 1966 but there was a decline in the contribution toward total fertility in 1967 and 1968. Age-group 45-49 shared the trend of age-group 40-44; that is, the percentage contribution of age-group 45-49 increased for 1961 to 1967 relatively to 1960, but there was a decrease in the contribution by this age-group in 1968.

So far we have discussed changes in the age-specific fertility pattern of U.S.A. females from 1960 through 1968 (Table 4.2). What needs to be considered now are changes in the relative contributions made by each age-group. Because the contribution of age-group 10-14 toward total fertility is very small, the age-group could experience extreme variations between 1961 and 1968 without much impact on total fertility. The index of relative composition for age-group 10-14 varies from a minimum of 5.5 per cent to 84.40 per cent. Although the relative contribution by age-group 10-14 almost doubled from 1960 to 1968, it remained quite insignificant. The other age-groups which experienced significant changes are 15-19 and 45-49.

The percentage distributions (Table 4.1) of fertility rates from 1960 through 1968 show that up to 90 per cent fertility in any year occurs to four age-groups: 15-19, 20-24, 25-29, and 30-34. Out of these four age-groups, age-groups 20-24 and 25-29 contribute most to the total fertility. Although the index of relative composition is the highest for age-group 10-14, the indices of dissimilarity for the years 1961 to 1968 are highly dominated by the age-groups 20-24 and 25-29. However, age-groups 20-24 and 25-29 behave in opposite directions. Percentage contribution by age-group 20-24 toward the total fertility decreased all the way from 1960 to 1968, while percentage contributions by the age-group 25-29 increased during the period 1960 to 1968. It may be concluded that the fertility decline from 1960 to 1968 was age-selective dominated by the decline in the contribution of the age-group 20-24 toward the total fertility during the period; though such a decline is compensated for by smaller increases in contributions of numerically less important age-groups. Such a shift in the percentage contributions flattened the fertility distributions of 1961 to 1968.

The index of dissimilarity in Table 4.2 shows the overall changes in the fertility schedules of 1961 to 1968 from the fertility schedule of 1960. For example, the changes from 1960 fertility pattern are different for 1966 from those of 1968. In 1966, the proportionate contributions of age-specific fertility rates toward the respective total

fertility increased for all age-groups but the 20-24. In 1968, the percentage contribution toward total fertility increased only for age-groups 10-14, 15-19, and 25-29. The rest of the reproductive age-groups show decline in their percentage contributions. In other words, the increase in percentage contributions by three age-groups is compensated for by the five age-groups in 1968, while in 1966 the decline in one age-group (20-24) is compensated for by the increase in the rest of the seven age-groups.

Although decline in U.S. fertility from 1960 to 1968 is mainly due to the fertility performance of age-group 20-24, age-group 20-24 remains the modal age-group. Changes in the fertility behaviour for the U.S. of other age-groups, particularly that of younger and older reproductive age-groups, are relatively unimportant.

Intrinsic Characteristics of Fertility Schedules

To see the long-range effects of the fertility schedules of 1960 through 1968, some demographic measures at the end of the process of stabilization for each fertility schedule are calculated. As the characteristics of the eventual stable population are, in fact, the characteristics of the demographic conditions imposed by us at the start of the process of stabilization, the demographic measures calculated at the end of the process are termed as intrinsic characteristics of the fertility schedules (U.N., 1968, pp. 1-10).

Table 4.4 summarizes the effects of the fertility schedules. It is clear from Table 4.4 that there is a monotonic decline as measured by crude birth rate and λ_1 in U.S.A. fertility from 1960 to 1968. It will be recalled from Chapter II that the values of λ_1 ($r = \log \lambda$) and the crude birth rate for females show yearly reduction in fertility from 1960 to 1968. The crude birth rate for females declined from 26.06 per one thousand in 1960 to 16.79 in 1968. The effects of the decline in fertility are felt on the composition of the final age-distributions. The percentage points for age 15 and below dropped from 34.82 for the 1960 schedule to 24.11 for the 1968 schedule while those for age 45 and above increased from 22.27 for 1960 to 34.77 for 1968.

Besides λ_1, the inertia of population is computed for each fertility schedule. The coefficient of population inertia has been defined by Bourgeois-Pichat (1971, p. 241) as the ratio of the size of a population at time t to the size of the same population at time t+n where n is the time interval. The coefficient of population inertia represents relative change in the size of a population at two points of time. Both the population inertia and λ_1 are interrelated concepts in that both are relevant to growth potential. While population intertia shows relative change in the size of a population, λ_1 is the dominant characteristic root. After a population has achieved stability,

TABLE 4.4

Intrinsic Characteristics for Female Population as a Result of the Fertility Schedules of U. S. A., 1960 - 1968

Characteristics	FERTILITY SCHEDULES								
	1960	1961	1962	1963	1964	1965	1966	1967	1968
λ_1	1.113	1.111	1.101	1.092	1.084	1.065	1.052	1.039	1.032
Population Inertia	45.84	31.77	43.30	34.94	16.04	10.45	7.61	5.67	3.93
Crude birth rate for Females	26.06	25.87	24.71	23.64	22.66	20.47	18.96	17.59	16.79
Percentage below 15 years of age	34.82	34.61	33.34	32.14	31.03	28.50	26.72	25.08	24.11
Percentage age 15-44	42.90	42.90	42.89	42.83	42.73	42.35	41.95	41.46	41.12
Percentage age 45+	22.27	22.48	23.77	25.03	26.24	29.15	31.34	33.46	34.77
Stabilization Time (year)	195	175	205	210	185	190	200	215	200

the values of λ_1 and population inertia are exactly the same for any two points of time.

In our analysis, we have computed the population inertia by taking the ratio of the size of the population at the time of stabilization to the size of the initial population. Table 4.4 shows that by the time a population achieves stability, the population with 1960 fertility would be 45.84 times the initial population, 31.77 times the initial population with 1961 fertility, 43.3 times with 1962 fertility, and so on. One could easily see the advantages of lower fertility from Table 4.4. Except for 1962, the population inertia decreases with the decline in fertility. The exception for 1962 fertility is due to the fact that with 1962 fertility the population stabilizes after 205 years of projection while with 1960 and 1961 fertility schedules the stability is achieved in 195 and 175 years, respectively. Although the population with 1963 fertility schedule stabilizes in 210 years, yet the fertility level of 1963 is of such a magnitude that the population is less than that of 1962 fertility.

INITIAL AND STABLE AGE-DISTRIBUTIONS FOR U.S.A., 1960-1968

The Problem

There are three determinants of each age-distribution: (a) fertility and mortality schedules, (b) age composition of population, and (c) the interaction

between age composition and the schedules of fertility and mortality (Coale, 1965). It is well recognized that the stable age-distribution is largely determined by the intrinsic rate of growth and the fraction of population p(a) which survives from age zero to a. Since we are using only one mortality schedule, one initial age-distribution, and a number of fertility schedules in our analysis, mortality and initial age compositional effects are controlled. Thus the stable age-distributions, in this analysis, are the result of the fertility schedules. Ryder (1960) has shown the effects of interaction between fertility distribution and the age composition of a population on the fertility rates. For example, Ryder (1960) has demonstrated that if the mean age of child-bearing falls, the fertility rates would increase,and the fertility rates would decrease if the mean age of child-bearing rises.

From Coale's and Ryder's works, one can conclude that out of the three determinants of age, fertility, and the interaction between fertility and age composition are vitally important. Concerning visualization of the process of stabilization, Coale (1972, p. 104) observes:

> A very useful point in such a visualization is recognition of this fact: There must be an initial population that would generate exactly the same exponential birth sequence as any given initial population--without any deviations or fluctuations. This population has a stable age-distribution from age 0 to and a total size to age β [$(Q\ /b)_0 \int^{\beta} C(a)\, da$, where C(a) is the proportion at age a in the stable population] such that the magnitude of its exponential birth sequence is the same as

> that of the given population. If this population is subtracted from the given initial age distribution up to age β, the remainder generates exactly the same deviations as the whole initial population--but a zero exponential sequence.

Where the deviations or fluctuations between the initial and the stable age-distributions are important it is also important to study the properties of these deviations. The following pages are an attempt to explain the properties of deviations. The questions as to how the initial population forgets its past age distribution, how much it differs from its stable distribution, and the probable reasons behind the differences between the two distributions form the core of this section.

Analysis

As mentioned earlier, the U.S.A. 1963 population is treated as the initial population. The age-specific fertility rates for the U.S.A. from 1960 to 1968 are applied to the initial population to obtain the stable age-distributions for each fertility schedule. In all, there are nine stable age-distributions and one initial age-distribution. Table 4.5 demonstrates the initial distribution and nine stable age-distributions, one stable population for each of the fertility schedules.

The differences between the initial age-distribution and each of the stable age-distributions are depicted in Table 4.6. Each of the stable age-distributions is

subtracted from the initial age-distribution to get the percentage point differences for each age-group.

The examination of Table 4.6 suggests that the index of dissimilarity (Δ) decreased from 1960 to 1962. Age-groups 0-4 to 30-34 are the ones which gain the proportions at the time of stabilization, while age-groups 35-39 and above lose the proportions for 1960, 1961, and 1962 fertility schedules. With the 1963 and 1964 fertility schedules, the age-group 85+ (oldest of all) joined the younger age-groups (0-4 to 30-34) in increasing their proportions. With 1965 fertility, the pattern of gains or losses in the proportions by age-groups is changed. The young age-groups 0-4, 5-9, and 10-14 and age-groups 35-39 to 70-74 lose while the rest of the age-groups gain proportionally. With the application of 1966 fertility rates, the age-groups 0-4 to 15-19 and the age-groups 30-34 to 55-59 lose proportionally while all other age-groups gain. With 1967 fertility, the age-groups 0-4 to 15-19 and the age-groups 35-39 to 50-55 lose and the age-groups 20-24 to 30-34 and 55+ gain in proportions. Age-groups 0-4 to 15-19 and 35-39 to 45-49 lose in proportions while the rest of the age-groups gain in proportion with 1968 fertility schedule.

On the basis of Table 4.6, it may be suggested that for higher fertility schedules, the younger age-groups gain proportions and the older age-groups lose in proportions

TABLE 4. 5

Percentage Distributions for the Initial Population and for the Stable Populations

For Females of U. S. A., 1960 - 68 Fertility Schedules

		STABLE AGE-DISTRIBUTIONS BY FERTILITY SCHEDULE								
Age-Group	Initial Population	1960	1961	1962	1963	1964	1965	1966	1967	1968
0 - 4	10.61	12.91	12.81	12.24	11.71	11.22	10.14	9.39	8.71	8.32
5 - 9	10.29	11.55	11.49	11.07	10.68	10.31	9.48	8.89	8.35	8.03
10 - 14	9.24	10.36	10.32	10.03	9.75	9.49	8.88	8.44	8.02	7.76
15 - 19	8.00	9.29	9.26	9.08	8.91	8.73	8.32	8.00	7.69	7.50
20 - 24	6.56	8.32	8.31	8.22	8.12	8.03	7.78	7.58	7.37	7.24
25 - 29	5.75	7.44	7.44	7.43	7.40	7.37	7.27	7.17	7.06	6.99
30 - 34	5.99	6.65	6.66	6.71	6.74	6.76	6.79	6.78	6.76	6.73
35 - 39	6.55	5.93	5.95	6.04	6.12	6.19	6.32	6.40	6.45	6.47
40 - 44	6.54	5.27	5.29	5.42	5.54	5.64	5.87	6.01	6.13	6.19
45 - 49	5.97	4.65	4.68	4.83	4.98	5.11	5.41	5.61	5.79	5.90
50 - 54	5.45	4.07	4.10	4.27	4.44	4.59	4.95	5.20	5.43	5.56
55 - 59	4.75	3.52	3.55	3.73	3.91	4.07	4.47	4.75	5.02	5.18
60 - 64	4.11	2.98	3.01	3.19	3.37	3.54	3.95	4.26	4.55	4.73
65 - 69	3.52	2.44	2.47	2.64	2.81	2.98	3.38	3.69	3.99	4.18
70 - 74	2.92	1.90	1.92	2.07	2.23	2.38	2.75	3.04	3.32	3.50
75 - 79	2.00	1.35	1.37	1.49	1.62	1.74	2.04	2.29	2.53	2.69
80 - 84	1.11	0.82	0.84	0.92	1.00	1.08	1.30	1.47	1.65	1.76
85+	0.64	0.55	0.56	0.62	0.68	0.74	0.90	1.04	1.18	1.27
Total	100	100	100	100	100	100	100	100	100	100

TABLE 4. 6

Percentage Point Differences Between Initial (1963) and the Stable Female Age-Distributions by Age-Groups and by Fertility Schedules

FERTILITY LEVEL Age-Group	1960	1961	1962	1963	1964	1965	1966	1967	1968
0 - 4	-2.30	-2.20	-1.63	-1.10	-0.61	0.47	1.22	1.89	2.29
5 - 9	-1.27	-1.20	-0.78	-0.39	-0.02	0.81	1.40	1.94	2.26
10 - 14	-1.12	-1.07	-0.79	-0.51	-0.25	0.36	0.80	1.22	1.48
15 - 19	-1.29	-1.26	-1.08	-0.90	-0.73	-0.32	0.00	0.31	0.50
20 - 24	-1.76	-1.75	-1.66	-1.57	-1.47	-1.23	-1.02	-0.82	-0.69
25 - 29	-1.70	-1.70	-1.68	-1.66	-1.63	-1.53	-1.43	-1.32	-1.24
30 - 34	-0.66	-0.67	-0.72	-0.75	-0.77	-0.80	-0.79	-0.77	-0.74
35 - 39	0.62	0.60	0.51	0.43	0.36	0.23	0.15	0.10	0.08
40 - 44	1.27	1.25	1.12	1.00	0.89	0.67	0.53	0.41	0.34
45 - 49	1.32	1.30	1.14	0.99	0.86	0.56	0.36	0.18	0.08
50 - 54	1.38	1.36	1.18	1.02	0.86	0.51	0.26	0.03	-0.11
55 - 59	1.23	1.20	1.02	0.84	0.68	0.28	0.00	-0.27	-0.43
60 - 64	1.13	1.10	0.92	0.74	0.57	0.16	-0.15	-0.44	-0.62
65 - 69	1.08	1.05	0.88	0.71	0.54	0.14	-0.17	-0.47	-0.66
70 - 74	1.03	1.00	0.85	0.70	0.55	0.18	-0.11	-0.40	-0.58
75 - 79	0.65	0.63	0.50	0.38	0.26	-0.05	-0.29	-0.54	-0.69
80 - 84	0.29	0.28	0.20	0.11	0.03	-0.18	-0.36	-0.54	-0.65
85 +	0.09	0.09	0.03	-0.04	-0.10	-0.26	-0.39	-0.53	-0.62
Index of Dissimilarity = Δ	10.10	9.85	8.34	6.92	5.58	4.37	4.72	6.08	7.03

as it is evident from the stable distributions of 1960, 1961, and 1962 fertility schedules. Since fertility declines every year from 1960 to 1968, the older age-groups gain and the younger age-groups lose proportionally. It is evident from the table that the first age-group which is affected by a reduction in fertililty level at the time of stabilization is the oldest age-group(s). (For example, with 1963 and 1964 fertility schedules, age-group 85+ gained proportionately to the initial age-distribution.) It will be remembered that the stable population theory tells us that the initial age-distribution and migration component have no effect whatsoever on the stable age-distribution. As the fertility is further reduced the next older age-group(s) gain in proportions. This can be seen from the percentage point difference distributions for each fertility schedule. With 1965 fertility level (which is lower than that of 1963) age-groups 75-79, 80-84, and 85+ gain in proportions. With 1966 fertility schedule, age-groups 60-64 and above gain proportionally; with 1967 fertility schedule, age-groups 55-59 and above, and with 1968, 50-54 and above gain in proportions.

What happens to the younger age-groups? The younger age-groups gain with higher fertility level and lose in proportions with lower fertility level by the time stabilization has been achieved. Proportional gain by age-group 0-4 with the 1960 schedule is maximum when compared

with the gains by 0-4 age-group in all the other fertility schedules. With the decline in each subsequent fertility level, the gains in proportions by age-group 0-4 become less and less until with the 1965 level they turn into actual losses. These losses increase monotonically until the 1968 level has been reached. The continuing dwindling in the proportions of this age-group is positively related with the reduction in fertility. Age-groups 5-9 through 25-29 behave identically, although the correlation gets weaker and weaker.

It is interesting to see that the proportionate differences between the initial and the stable age-distributions have varying patterns. With 1960 through 1962 fertility schedules, the young age-groups (0-4 to 30-34) gain in proportions while the older age-groups (35-39 and above) lose. With 1963 fertility, the process of gains or losses in the proportion changes so that the oldest age-groups start gaining and the youngest age-groups losing. The pattern of positive and negative signs with 1960, 1961, and 1962 fertility schedules is upset by 1963 fertility. There is no more consistent demarcation of positive and negative signs. The fact remains that with lower fertility, the older age-groups gain in proportions in a descending order of age and the very young age-groups lose in an ascending order of age. One could probably anticipate that with further reduction of fertility from 1968 level, the pattern of positive and negative signs in percentage

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point difference distribution would form the pattern consistent with those of 1960, 1961, and 1962 fertility schedules. Such a pattern with lower fertility level would be opposite to that of higher fertility levels (Frejka, 1973, p. 103). For example, with a low fertility, all those age-groups that gain with high fertility schedules would lose in proportions and the older age-groups which lose in proportions with high fertility schedules would gain with low fertility schedules (Kiser, 1968, and Rosset, 1966).

In other words, low or high fertility produces older or younger populations by means of gains or losses in the proportions of age-groups. What about a fertility level through which either younger or older age-groups do not lose or gain in proportions? Or, if they do, the losses or gains in proportions at the extreme ages are negligible. Such a fertility level is possible in human populations as it is evident from the effects of 1965 fertility schedule on the age-distribution. Should such a population be classified as younger or older? A fertility level through which a population neither loses or gains proportions at the extreme ages can be classified as an 'adult' population for the given age-distribution and the given band of fertility schedules. In an adult population, the major changes in the proportions occur in the middle age-groups.

The distribution generated by 1965 fertility schedule with the 1963 age-distribution is close to such an 'adult' concept.

The understanding of the losses or gains in the proportions leads us to argue that the index of dissimilarity (Δ) for two different fertility levels may be the same but their distributions of gains and losses may well be entirely different. In our nine distributions of percentage point differences (of which Δ is an outcome), it is evident that high fertility (1960) has the largest value of Δ and that the value of Δ decreases monotonically from the 1960 through 1965 fertility schedules. From the 1966 through 1968 fertility schedules, Δ increases monotonically. With another couple of still lower fertility schedules, the Δ would presumably increase further and reach a level close to that of the 1960 fertility. The 1963 and 1968 fertility schedules give almost identical indices of dissimilarity, yet the distributions of gains and losses are entirely different. This shows that the Δ alone does not render full information and that the complete percentage point difference distribution from which Δ is derived is necessary to illuminate further the differences between the two distributions.

The index of dissimilarity has been used to state the distance of the initial age-distribution from the

stable age-distribution (Keyfitz, 1968, p. 47, and Mukherjee, 1973). Mukherjee (1973) suggests that the size of Δ can be used as a criterion to determine whether a population is stable or not. Out of different sets of initial and stable age-distributions, he treats one set as the standard one. The Δ computed for the standard distributions is termed the standard index of dissimilarity. By using the notion of standard index of dissimilarity, Mukherjee (1973) states that the age-distribution with smaller than the standard Δ be considered as stable and the distributions with larger Δ than the standard one as non-stable. However, a comparison of the last lines of our Tables 4.4 and 4.6 shows that the size of Δ is a poor indicator of the temporal distance between an initial age-distribution and its stable form.

The distance between the initial and the stable age-distributions as suggested by Keyfitz and Mukherjee may be interpreted in two ways: (1) How much time the initial age-distribution with given fertility and mortality schedules would take to reach stability, and (2) How much a stable distribution differs, in terms of proportions, from its initial age-distribution. However, and to repeat, there is no one-to-one consistency between these interpretations of the distance between initial and stable age-distributions. A detailed critical comment on Keyfitz's

and Mukherjee's argument is provided in the following chapter. On the other hand, Δ does show how much an initial population changes its age when it achieves stability if a given fertility schedule is kept constant. A smaller value of Δ suggests that the initial population was reasonably adult to begin with; that is, it has become neither older nor younger by the time of stabilization. In other words, the age of stable population is about the same as the age of the initial population.

CONCLUSION

The highlights of the present chapter may be summarized as:

1. Whereas the fertility of the U.S.A. declined from 1960 to 1968, there is also a change in the pattern of contribution by age-groups toward the total fertility. The relative contribution toward total fertility increased for younger age-groups (for example, 10-14), decreased for older ages, but the pattern of fertility remained dominated by age-groups 20-24 and 25-29.

2. In the percentage contribution toward total fertility, age-group 20-24 showed a decline from 1960 to 1968 but it remained the modal age-group, while the age-group 25-29 showed an increase in the percentage contribution toward total fertility from 1960 to 1968.

3. Contrary to the suggestion of Keyfitz and Mukherjee, the index of dissimilarity does not suggest how distant, in the temporal sense, an initial age-distribution is from its stable distribution. Rather, the index shows the extent to which a population has changed its age during the process of stabilization. As an adjunct of percentage point difference distribution, the index of dissimilarity is a useful measure in showing how much older or younger a population becomes relatively to its initial age after a given fertility schedule remains constant.

4. With narrow changes in our empirical fertility the very young and the very old age-groups of our one distribution are affected. The middle age-groups of our one distribution are not as sensitive as the extreme age-groups in terms of gains or losses in the proportions. However, we have not considered, in this thesis, whether other age-distributions and different families of fertility schedules would produce different results, except for one used of Japanese data in Chapter V.

CHAPTER V

DISTANCE BETWEEN INITIAL AND STABLE AGE-DISTRIBUTIONS

In Chapter IV, we have found that the index of dissimilarity for a given age-distribution is a function of changes in fertility. We also found that the Δ indicates the changes in the age of an initial population after the stabilization process has been completed. The large or small value of Δ, with the aid of the proportionate distribution, shows how much an initial population would be younger or older at the time it achieves stability with a given fertility level. The size of Δ is predominantly affected by the very young or very old age-groups. However, our conclusions in this regard were based on only one age-distribution and a rather narrow range of nine fertility schedules. In this chapter, the arguments about the size of Δ and the distance between the initial and the stable age-distributions are continued and extended by examining the effects of a high fertility schedule and a below-replacement level of fertility schedule on the initial age-distributions.

The notion of distance is widely used in mathematics and mathematical statistics, mainly in the analysis of topological spaces (Rudin, 1966, p. 9).

Distance function is non-negative in nature. Index of dissimilarity may be classified as equivalent to the notion of absolute distance function. In this chapter, the distance between the initial and the stable age-distribution means either (a) the proportionate difference between the initial and the stable age-distribution or (b) how much time an initial population with given fertility and mortality schedules takes to achieve stability.

ANALYSIS

The 1963 age-distributions for the U.S.A. and Japan and their respective fertility schedules are selected. In all, four stable age-distributions, namely, A, B, C, and D are sought. In distribution A, the U.S.A. 1963 fertility and age-distribution interact to produce a stable age-distribution. Stable age-distribution B is an outcome of the U.S.A. 1963 fertility schedule and Japan's 1963 initial population. In distribution C, Japan's fertility (U.N., 1963) and her 1963 age-distribution, and in D, Japan's fertility and the U.S.A. 1963 age-distribution interact, respectively.

The interaction between the age-distributions and fertility schedules is summarized in the following form:

Interaction between Fertility Schedules
and Age-Distributions*

AGE DISTRIBUTION	FERTILITY	
	U.S.A. 1963	JAPAN 1963
U.S.A., 1963	A	D
JAPAN, 1963	B	C

*Alphabetical characters in the cells are the resultant stable age-distributions.

All the combinations of fertility and initial age-distributions are exposed to the U.S.A. 1963 mortality schedule. Such an interaction between fertility and initial age-distributions aims at explaining the impact of the initial age-distributions (Keyfitz and Flieger, 1971, pp. 20-35) on the process of stabilization of a population.

Table 5.1 shows the initial age-distributions for the U.S.A. 1963 and Japan 1963 (columns 2 and 3) along with the percentage point differences between the two in column 4. In columns 5 through 8, the stable age-distributions by each combination of fertility and initial age-distribution are presented. There are two identical stable age-distributions for each fertility schedule. Since these two stable age-distributions have different initial age-distributions, the indices of dissimilarity for the identical stable distributions must be different.

TABLE 5.1

Initial and Stable Female Age-Distributions by Fertility

Schedule and Initial Age-Distributions

	X	Y	X-Y	Stable Age-Distributions as a Result of			
Age-Group	U.S.A. 1963 Age-Distribution	Japan 1963 Age-Distribution	% Difference	A U.S. Fertility U.S. Age-Distribution	B U.S. Fertility Japan Age-Distribution	C Japan Fertility Japan Age-Distribution	D Japan Fertility U.S. Age-Distribution
0-4	10.61	7.84	2.77	11.71	11.71	6.12	6.12
5-9	10.29	7.85	2.44	10.68	10.68	6.17	6.17
10-14	9.24	9.64	-0.37	9.75	9.75	6.23	6.23
15-19	8.00	10.29	-2.29	8.91	8.91	6.28	6.28
20-24	6.56	9.49	-2.93	8.12	8.12	6.34	6.34
25-29	5.75	8.38	-2.63	7.40	7.40	6.38	6.38
30-34	5.99	8.14	-2.15	6.74	6.74	6.42	6.42
35-39	6.55	7.44	-0.89	6.12	6.12	6.45	6.45
40-44	6.54	6.40	0.14	5.54	5.54	6.44	6.44
45-49	5.97	5.25	0.72	4.98	4.98	6.41	6.41
50-54	5.45	4.97	0.48	4.44	4.44	6.31	6.31
55-59	4.75	4.05	0.70	3.91	3.91	6.14	6.14
60-64	4.11	3.45	0.66	3.37	3.37	5.86	5.86
65-69	3.52	2.65	0.87	2.81	2.81	5.40	5.40
70-74	2.92	1.89	1.03	2.23	2.23	4.73	4.73
75-79	2.00	1.27	0.73	1.62	1.62	3.79	3.79
80-84	1.11	0.69	0.42	1.00	1.00	2.60	2.60
85+	0.64	0.34	0.30	0.68	0.68	1.95	1.95
	100%	100%	$\Delta = 11.26$	100%	100%	100%	100%

It is evident from Table 5.1 that the initial age-distribution of Japan is an unusual one with the first half of the age-distribution resulting from the low fertility of the immediate past and the second half from the high fertility of the distant (assuming no migration) past. The initial age-distributions suggest that Japan had lower birth rates from 1953 to 1963 than the U.S.A.. In fact, the high proportions in the age-distribution of Japan start from age-group 10-14. This means that Japan's fertility before 1953 was higher than the fertility during the period 1953 to 1963. Proportions in the age-distribution of Japan are higher than in the U.S. age-distribution between age-groups 10-14 through 35-39. For other age-groups, the U.S.A. distribution has proportions greater in size than those of Japan's distribution.

Table 5.2 demonstrates the index of dissimilarity and the percentage point differences between the initial and the stable age-distribution for each of the sets of fertility (A, B, C, and D). In set A, the fertility and the age-distribution of the U.S.A. 1963 interact to produce a stable age-distribution. The index of dissimilarity for set A is 6.92. It will be remembered that this set appears in Table 4.6, the 1963 column. Losses and gains in the proportions by the stable age-distribution from the initial age-distribution show that the older age-groups except 85+ lost while the younger age-groups gained proportionally,

TABLE 5. 2

Index of Dissimilarity and the Percentage Point Differences for Different Combinations of Initial Female Populations and Fertility Schedules

Age-Group		Proportionate Differences Between Initial and Stable Age-Distributions of Combinations			
		A	B	C	D
	Fertility	U.S.A.	U.S.A.	Japan	Japan
	Age-Distribution	U.S.A.	Japan	Japan	U.S.A.
0 - 4		-1.10	-3.87	1.72	4.49
5 - 9		-0.39	-2.83	1.68	4.12
10 - 14		-0.51	-0.15	3.38	3.01
15 - 19		-0.90	1.38	4.01	1.72
20 - 24		-1.57	1.37	3.16	0.22
25 - 29		-1.66	0.97	2.00	-0.64
30 - 34		-0.75	-1.40	1.71	-0.43
35 - 39		0.43	-1.32	1.00	0.11
40 - 44		1.00	0.87	-0.04	0.09
45 - 49		0.99	0.27	-1.16	-0.43
50 - 54		1.02	0.53	-1.34	-0.86
55 - 59		0.84	0.15	-2.09	-1.39
60 - 64		0.74	0.08	-2.40	-1.75
65 - 69		0.71	-0.17	-2.75	-1.88
70 - 74		0.70	-0.34	-2.84	-1.80
75 - 79		0.38	-0.35	-2.52	-1.79
80 - 84		0.11	-0.31	-1.91	-1.48
85 +		-0.04	-0.33	-1.60	-1.30
Index of Dissimilarity		6.92	8.35	18.65	13.95

signifying that 1963 U.S.A. fertility eventually produces a younger distribution.

The interaction between U.S.A. fertility schedule and Japan's age-distribution in set B produces Δ of 8.35. The proportionate difference between the initial and the stable age-distributions suggests that age-groups 0-4, 5-9, 10-14, and 65+ gained while the other age-groups lost proportionally. The three young age-groups shared almost 82 per cent of the total gains in proportions while the five older age-groups contributed the remaining 18 per cent. It shows that the Japanese population, if it adopts U.S. fertility of 1963, would eventually be a relatively younger population than it was in 1963.

The interaction between Japan's 1963 fertility rates, which are below replacement level, and the U.S.A. 1963 age-distributions (D) produces an older population. The young age-groups 0-4 to 20-24 lost while the older age-groups gained proportionally. The proportions lost by age-groups 35-39 and 40-44 are very small; that is, they constitute only 1 per cent of the total losses. Had the fertility level been a little lower, age-groups 35-39 and 40-44 would have gained, thus leaving all proportional losses to the younger age-groups.

The percentage point difference distribution (C) is a result of Japan's fertility schedule and Japan's age distribution. It shows a consistent pattern of proportional

losses and gains; that is, losses occur to all younger age-groups while all older age-groups gain. Out of A, B, C, and D sets the index of dissimilarity is maximum for C and minimum for A. The value of Δ for A is 6.92; for B, 8.35; for C, 18.65; and for D. 13.95.

DISTANCE BETWEEN INITIAL AND STABLE POPULATION IN TERMS OF PROPORTIONS

Let us examine Keyfitz's (1968) and Mukherjee's (1973) argument that the size of Δ is an indication of the distance between initial and stable age-distribution. The distance as defined earlier may be interpreted alternatively in terms of proportions or the time taken by a population to achieve stability. In the sets A, B, C, and D, we have seen that the interactions between the same fertility schedule and different initial age-distributions produce indices of dissimilarity of different sizes. It is an axiom of stable population theory that with fixed mortality the eventual age-distribution is the eventual effect of the present fertility, while the initial age-distribution is the effect of fertility in the past.

What the index of dissimilarity gives us, in fact, is the difference between the effects of the past and the present fertility. If the initial distribution is free of migration effects and the fertility of the past and present remained the same, the size of Δ would be equal to zero. If fertility of the past is higher than the

present fertility, the initial age-distribution which is the result of the past fertility would be younger in age. The stable age-distribution which is a result of the present (low) fertility would be older in age. Sum total of the differences between stable and initial age-distributions of the similar signs, Δ , would be large. Similarly, an initial age-distribution which is older in age and a younger stable age-distribution would produce a large Δ. The size of the index of dissimilarity may be presented in the following form:

Size of Δ	Initial Population Level of Age	Stable Population Level of Age
Large	Young	Old
Large	Old	Young
Small	Young	Young
Small	Old	Old
Small	Adult	Adult
Medium	Adult	Old
Medium	Adult	Young
Medium	Old	Adult
Medium	Young	Adult

As we have stated in Chapter IV, the Δ along with the distribution of the proportionate differences between the initial and the stable population tells us about the aging process of a population when it converges to stability. A large Δ shows that the age of stable population is entirely different from the initial

population; a small value of Δ signifies little change in the age of the population and a medium size Δ shows a moderate change in age of a population at the time stability is achieved.

So far, we have found that similar fertility schedules with dissimilar initial age-distribtuion or similar age-distributions with different fertility schedules do not have the same value of the index of dissimilarity. To explain the differences in the values of the index of dissimilarity, we explored the possibility of examining the effects of fertility and the initial age-distribution separately on the stable age-distribution. As U.S.A. 1960 to 1968 fertility schedules were applied to only one initial population (U.S.A. 1963 age-distribution), one cannot make use of the nine stable age-distributions. Instead the examination of the A, B, C, and D sets of fertility and age-distributions serves our purpose.

Pairs of A, B, C, and D that have one of the two things (fertility or initial distribution) in common are formed as AB, AD, BC, and CD. Pair AB has the same fertility (U.S.A., 1963) but different initial age-distributions. The difference between the values of Δ in A and B is (6.92-8.35) 1.43 (ignoring the sign). This difference is divided by the larger value of Δ in the pair. The multiple of 100 would give us the percentage contribution of the initial distribution towards the

dissimilarity of the stable distribution. The same process may be employed in evaluating the effects of fertility. The following percentages are the product of this approach.

Pair	Difference in Δ	Difference in Δ Due to
AB	17.12%	Age-distribution
AD	50.39%	Fertility
BC	55.22%	Fertility
CD	25.20%	Age-distribution

In pairs AB and AD, 17.12 per cent and 50.39 per cent differences are explained as the differences due to age structure and fertility, respectively. In pairs BC and CD, 55.22 and 25.20 per cent differences in Δ are due to fertility and age structure, respectively.

LENGTH OF THE PROCESS OF STABILIZATION

We saw that the Δ is an indicator of the distance of initial population from its stable form in terms of proportions, but the available evidence discussed above has shown that it is a poor indicator of temporal distance. We have seen in the preceding pages that the similar values of the Δ (1963 and 1968 in Table 4.6) can be the result of different combinations of past and present fertility. The other aspect of the distance of initial population from its stable form is the length of the

process of stabilization; that is, how much time the initial population, with given schedules of fertility and mortality, takes to reach stability.

The length of the process of stabilization has been discussed in literature. Coale (1972, p. 92) has developed mathematical expressions for the coefficients of the various terms in the birth sequence as a function of the initial age-distribution. These coefficients are determined by the net fertility schedule. For the different initial age-distributions that are subject to the same net fertility function, the magnitude of the coefficients in the birth sequence are analysed by Coale (1972, pp. 100-104) to examine the initial circumstances that lead to the deviations from the exponential birth sequence and the rate at which a population converges to its stable form. It has been observed that the small deviations from the exponential birth sequence imply rapid convergence toward stability while the large deviations imply slow convergence.

Keyfitz (1972, pp. 1-38) has made use of the first four moments of the net maternity function to generalize about the time of stabilization. In Keyfitz's words:

> . . . The convergence will be more rapid the greater the positive amount of the third moment: skewness on the right hastens convergence. The fourth moment acts in the opposite direction to σ^2 in the numerator and in the denominator: negative kurtosis helps convergence (1972, p. 10).

Coale's and Keyfitz's analyses of the rate of convergence to stability are extensive in mathematical treatment but too technical to follow. In the present section, an attempt is made to explain the length of the process of stabilization using the notion of interaction between the initial populations and the fertility schedules.

Table 5.3 shows that for the U.S.A. fertility schedules for 1960 through 1968, the time taken by initial age-distribution to achieve stability as defined in Chapter II does not seem to be related with either fertility or with indices of dissimilarity. Time for stabilization is maximum for the 1967 and 1963 fertility schedules. The initial population takes least time to stabilize with 1961 and 1964 fertility schedules. However, it is difficult to determine the relationship between time taken to achieve stability and the fertility schedules.

Relying exclusively on the one selected U.S.A. age-distribution, one gets meagre results. As evident from Table 5.3, fertility does not have any consistent relationship with the length of the process of stabilization. Since mortality is also kept constant for all age-distributions, we are left with only the analysis of the initial and stable age-distributions.

For the four sets of the interaction between fertility and initial age-distribution (A, B, C, and D), the time taken in achieving stability is 210 years for

TABLE 5.3

Length of the Process of Stabilization for U.S.A., Fertility Schedules 1960-1968 by the Index of Dissimilarity, Using U.S. 1963 Female Age-Distribution as Initial Population

Fertility Schedule	Index of Dissimilarity [1]	Length of the Process (in years) [2]
1960	10.10	195
1961	9.85	175
1962	8.34	205
1963	6.92	210
1964	5.58	185
1965	4.37	190
1966	4.72	200
1967	6.08	215
1968	7.03	200

[1]Source: Bottom row of our Table 4.6

[2]Source: Bottom row of our Table 4.4

set A, 230 years for B, 350 years for C, and 390 years for D (Table 5.4). As discussed earlier in this chapter, sets A and B have the same fertility but dissimilar initial age-distributions; also C and D have the same fertility but dissimilar initial age-distributions. Sets A and D have different fertility but the same age-distribution, while B and C have the same age-distribution but different fertility schedules.

What we do know is that the foremost task of a given fertility schedule in the early years of projections or during the early part of the process of stabilization is to remove the oscillations inherent in the age-distribution due to past fertility changes, past age-misreparting and past migration (Chapter III). After the severe oscillations are overcome, the process of stabilization is rapid. Such a reasoning is supported by the length of the process of stabilization for sets A, B, C, and D from Table 5.4.

It can be demonstrated graphically that the crude birth rate for females during the process of stabilization fluctuates for a longer period of time for the set B where U.S.A. fertility is applied to Japan's age structure and set D where Japan's fertility is applied to U.S.A. age structure. The fluctuations in the crude birth rate disappear in sets A and C earlier than in B and D. In the sets A and C, the combination of fertility and age-structure

TABLE 5.4

Length of the Process of Stabilization by Fertility Schedules and Initial Age Distributions (in Years)

AGE DISTRIBUTION	FERTILITY SCHEDULE U.S.A., 1963	JAPAN, 1963
U.S.A., 1963	210	390
JAPAN, 1963	230	350

is from the same population; that is, in set A, the U.S.A. fertility and U.S.A. age-structure interact and in set C Japan's fertility and Japan's age-structure.

Three conclusions can be drawn from the preceding discussion: (1) Low fertility schedules take a longer time to stabilize than high fertility schedules, (2) Fertility schedule foreign to the initial age-structure takes relatively more time to achieve stability than the fertility schedule which is a product of the initial age-structure, and (3) A low fertility schedule interacting with an initial population that is foreign to the schedule takes the maximum time for stabilization than (1) or (2). For further elaboration of these conclusions, the following observations from Table 5.4 are made.

As we know, it is difficult to classify Japan's age-structure of 1963 as either young or old since it has smaller proportions at younger and at older ages and larger proportions at the reproductive ages (see Chapter IV). We also demonstrated in Chapter III (Figures, 3.2, 3.3, and 3.4) that low level fertility schedules take more time to smooth the distributions. The Japanese 1963 fertility schedule of below replacement level takes a relatively longer time to smooth out the inconsistencies in the initial age-structure (sets C and D in Table 5.4).

In set D, the time taken for stability is 390 years, which is longer than that of C. In the case of set D,

fertility remains the same as of C (Japan's fertility) but the initial age-structure is that of U.S.A.. Thus, the longer process of stabilization for set D than C is due to both fertility below replacement level and a foreign age-structure in set D, while in set C only fertility of negative growth potential is the cause of the lengthy process. Set B takes more time to stabilize than set A. Since U.S. 1963 fertility is used in sets A and B, which is relatively of higher magnitude than that of sets C and D (Japan's 1963 fertility), it takes less time to overcome the oscillations in the initial age-structure and hence a shorter time for stabilization than C and D. The differences in the time of stabilization between A and B are due to the 'native' and 'foreign' age-structure, respectively, as are those between C and D.

We can now attempt to throw some light on the inconsistent pattern of the length of the process of stabilization, in Table 5.3, keeping in mind all the time that the nine stable populations there arise out of <u>one</u> initial age-distribution. As we know that the U.S.A. 1963 population is, in fact, not a closed population, the initial age-distribution used for the nine fertility schedules is distorted due to migration and other factors. Therefore, it is unrealistic to assume that 1963 fertility schedule of the U.S.A. is the 'native' and the rest are 'foreign' to the U.S.A. 1963 age-distribution. However, it may be noted that stabilization time for all fertility

schedules of the U.S.A. 1960-1968 lies in between 175 to 215 years, which shows that the time of stabilization varies with the degree of 'foreignness' of a fertility schedule from the initial age-structure.

CONCLUSION

We have dealt with the distance between initial and stable populations, both in terms of proportionate difference and in terms of time of stabilization. We have demonstrated in the present chapter that the size of the index of dissimilarity is a poor indicator, if it is an indicator at all, of the temporal distance. Our analysis shows that the proportionate differences are related with the aging process of a population.

The length of the process of stabilization is found to be related with (a) the level of fertility, (b) the degree of agreement between fertility and initial population, and (c) the interaction between (a) and (b). It is observed that low fertility schedules take a longer time to stabilize than high fertility schedules. The degree of 'foreignness' of a fertility schedule to the initial population is positively related with the time of stabilization. However, if a low fertility schedule interacts with a 'foreign' initial population, the time of stabilization is longer than that for the first two instances mentioned above.

CHAPTER VI

SUMMARY AND CONCLUSION

In this chapter summary of the major findings of the present research is presented and some suggestions are also made to promote the use of the stable population model in socio-demographic research.

MAJOR FINDINGS

The present study starts by reviewing models of mortality, fertility, and population growth. Some aspects of a stable population model are examined by means of the projection matrix approach. Questions such as how the stabilization process of a human population can be visualized, and how far is the initial population from its stable form, and how the aging process of a population takes place, are dealt with.

It must be emphasized, however, that our conclusions are based on limited observations. Data used in our study have limitations such as narrow range of fertility changes, one mortality schedule, and one initial age-distribution (except in Chapter V). Therefore, the results or conclusions of this thesis should be interpreted with caution.

The process of stabilization is examined by means of the notion of slope(s) (one of the several tools of

analysis) of an age-distribution. Slope of an age distribution is defined as the difference between the proportions of two successive age-groups. And it has been possible to demonstrate, theoretically and empirically, that the slope notion is both a valid and reliable criterion to study the stabilization process.

The mean and variance of the slope distribution are used to investigate the fertility effects on the stabilization process. It is noted that during the early period of population projections, any marked irregularities in the age-distribution inherited from the initial age-distribution are eliminated by fertility schedules. High fertility schedules overcome these irregularities in less time than low fertility schedules. At the start of the process of stabilization, the relationship between the variance of the slope distribution and the fertility level is negative but during the process this relationship diminishes. At the end of the process, the variance is related to the aging of the population.

The stabilization process simultaneously manifests the aging process of a population under given conditions of fertility and mortality. Excepting the extreme changes in mortality, it is well-documented in the literature that a higher fertility schedule eventually produces a younger population while a low fertility schedule produces an

older population. It is indicated in Chapter IV that the extreme age-groups (very young and very old) are the most responsive to the level of fertility. With a low fertility schedule, the young age-groups lose while the old age-groups gain proportionally; and with a high fertility schedule, the old age-groups lose and the young age-groups gain proportionally.

It has been demonstrated that, over a period of time, changes in the age-distribution of a population depend on the differences between the past and the present fertility. By past fertility we refer to the fertility which produced the initial age-distribution and the present fertility refers to the fertility which produced the stable age-distribution. If the level of the present and the past fertility is the same and the migration component is zero, there would be no change in the age-distribution of a population.

The discussion of the aging process of a population leads us to infer about the distance between the initial and the stable age-distributions. This distance is analysed by means of (1) the proportionate difference between the stable and the initial age-distributions, and (2) the time an initial population takes to achieve stability with given fertility and mortality schedules. In expressing the distance between the two distributions, the index of dissimilarity was used following Keyfitz (1968) and Mukherjee (1973). We have found that the index of dissimilarity is

a good indicator of the aging process of a population.

The time of stabilization is found to be related with the level of fertility and the consistency between the fertility schedule and the initial age-distribution. It is observed that high fertility schedules take less time to stabilize than low fertility schedules. Time of stabilization is also negatively related with the degree of consistency between the initial age-distribution and the fertility schedule. For example, a very young initial age-distribution with low fertility stabilizes over a longer period of time than an old initial age-distribution with low fertility.

SOCIOLOGICAL RELEVANCE OF THE STABLE POPULATION MODEL

The stable population model is of interest in technical demography especially in the areas of estimation of population parameters. The reason that this model has not been adopted by researchers in the study of social phenomena is owing to its abstract (and hypothetical) nature. While it is a truism to say that no population is stable in nature, studies of the effects of fertility on age-distribution, hence changes inherent in a population, are of interest in the formulation of policy on economic and social development.

The long- and short-term effects of the changes in the demographic phenomena affect the socio-economic aspects of a society in a variety of ways. Short-term effects of fertility have been discussed by Easterlin (1961, 1968), Akers (1968), Krotki (1969, 1970), Feeney (1970) and Keyfitz (1972). All these studies deal with the effects of postwar 'baby boom' on the life in the U.S.A. and Canada. The focus of these is generally on marriage market, labour market, social welfare, and social mobility, but they do signify how the demographic changes bring about disruptions, however unobtrusive, in the on-going process of social life.

Lack of information on the oscillations in the demographic phenomena can impede the effectiveness of social planning, because these oscillations are closely tied in with the other aspects of collective social life. A case in point is the report of the U.S. Commission on Population Growth and American Future (U.S., 1972, p. 110) about the feasibility of achieving an immediate zero population growth in the U.S.A.:

> . . . . Some called for zero growth immediately. But this would not be possible without considerable disruption to society. While there are a variety of paths to ultimate stabilization, none of the feasible paths would reach it immediately. Our past rapid growth has given us so many young couples that, even if they merely replaced themselves, the number of births would still rise for several years before levelling off. To produce the number of births consistent with immediate zero growth, they would have to limit their childbearing to an average of

> only about one child. In a few years, there would be only half as many children as there are now. This would have disruptive effects on the school system and subsequently on the number of persons entering the labor force. Thereafter, a constant total population could be maintained only if this small generation in turn had two children and their grandchildren had nearly three children on the average. And then the process would again have to reverse, so that the overall effect for many years would be that of an accordion-like continuous expansion and contraction.

It is useful to visualize the long-range effects of demographic variables for long-term socio-economic planning. For example, if the U.S.A. experiences the fertility schedule of 1960 for a long period of time, say 195 years, the population size would be almost 46 times the size in 1963. Such a figure looks more dreadful if one imagines that after 195 years from 1963 there would be 46 times more cities like New York or Chicago.

High fertility also poses economic problems. In the case of developing countries, for example, the impact of a population increase on educational development may be found in a variety of studies (Jones, 1969; Jones and Mitra, 1969; Jones and Kayani, 1971). Taken together, these pieces of research have indicated how the demographic variables can obstruct the attainment of educational goals in developing countries.

High fertility can have some repercussions on developed countries as well. For the U.S.A., Reed and McIntosh (1972) have estimated that the total discounted

and undiscounted cost of a first child is $59,627 and $98,361, respectively. From most of the studies on cost and benefit analyses one can infer that in a high fertility schedule, a nation has to spend a lot to meet the requirements of raising children both qualitatively and quantitatively.

Owing to the problems resulting from high fertility, developing countries are concerned with bringing down high growth rate in their populations. The developed countries such as the U.S.A. and Canada are heading toward the goal of zero population growth. For the U.S.A., the concern with population planning is accented by the Commission on Population Growth and American Future (U.S., 1972) recommending the goal of zero population growth in the near future. It has been argued by the Commission that the zero population growth would contribute significantly to the nation's ability to solve its problems associated with the quality of life. It may be noted that the Commission's recommendations are based primarily on the works of Frejka (1968, 1972) and Coale (1972a). Both of these authors have shown how a stationary population can be achieved and maintained and what kind of social benefits will accrue from the stationarity.

This poses a key dilemma before the designers of population policy. For effective population policy, a constant age-distribution is desirable. The low fertility will take longer time to produce a constant age-distribution

than the high fertility. On the other hand, high fertility will speed up population growth.

Several long-term advantages of low fertility are conceivable. However, a new set of problems associated with the aged people emerges as a by-product of the phenomenon of low fertility. For example, in the U.S.A. during the past few decades, public concern about the aged people has been expressed in terms of their increasing needs for money and health care (U.S., 1972). The U.S. Population Commission (1972) has pointed out the two sets of issues resulting from the increasing number of the aged people. The first set of issues involves matters of public ethics, personal preferences, and allocation of public expenditure. The second set of issues is concerned with the type of institutional care required for the growing number of the aged people in the population.

Another by-product indirectly stemming from low fertility but directly related with the presence of the aged people is the problem of their roles and statuses in society. There is an appreciable decrease in the role and status evaluation of the aged people. They become, more or less, non-participants in the life-sustaining activities of society. It is almost inevitable that the society of which these people are a part will have to take care of them. Alternatively, if the rate of fertility continues unhampered the society will be faced with a younger population and the

problems associated with the raising and up-bringing of younger people. Writers such as Ryder (1964), Riley, Johnson, and Fonex (1972) have demonstrated how the problems of the aged may become as serious as the problems of an increasing number of young people. Therefore, an increase or decline in fertility may be viewed from the societal perspective as to the desirability of having a large number of older people over the younger ones, or the other way around.

To sum up, the stable population model is a theoretically stringent model in that it is unlikely that any population could ever achieve a stable form. Thus, the model itself is restrictive but allows for envisioning possibilities of a "stable" population. To overcome the restrictiveness of the stable population model, demographers have developed various alternative conceptualizations. Most populations of the world would fall under one or the other of these conceptual formulations, hence their practical usefulness is undeniable. However, the usefulness of the stable population lies in its abstract nature. The model can serve as an umbrella under which the less abstract concepts such as 'near-stability,' 'semi-stability,' and 'quasi-stability' can be placed.

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